16

Electrical Energy and Capacitance

QUICK QUIZZES

1. Choice (b). The field exerts a force on the electron, causing it to accelerate in the direction opposite to that of the field. In this process, electrical potential energy is converted into kinetic energy of the electron. Note that the electron moves to a region of higher potential, but because the electron has negative charge this corresponds to a decrease in the potential energy of the electron.

2. Choice (a). The electron, a negatively charged particle, will move toward the region of higher electric potential. Because of the electron's negative charge, this corresponds to a decrease in electrical potential energy.

3. Choice (b). Charged particles always tend to move toward positions of lower potential energy. The electrical potential energy of a charged particle is $PE = qV$ and, for positively-charged particles, this decreases as V decreases. Thus, a positively-charged particle located at $x = A$ would move toward the left.

4. Choice (d). For a negatively-charged particle, the potential energy ($PE = qV$) decreases as V increases. A negatively charged particle would oscillate around $x = B$ which is a position of minimum potential energy for negative charges.

5. Choice (d). If the potential is zero at a point located a finite distance from charges, negative charges must be present in the region to make negative contributions to the potential and cancel positive contributions made by positive charges in the region.

6. Choice (c). Both the electric potential and the magnitude of the electric field decrease as the distance from the charged particle increases. However, the electric flux through the balloon does not change because it is proportional to the total charge enclosed by the balloon, which does not change as the balloon increases in size.

7. Choice (a). From the conservation of energy, the final kinetic energy of either particle will be given by

$$KE_f = KE_i + \left(PE_i - PE_f\right) = 0 + qV_i - qV_f = -q\left(V_f - V_i\right) = -q(\Delta V)$$

For the electron, $q = -e$ and $\Delta V = +1$ V giving $KE_f = -(-e)(+1\text{ V}) = +1$ eV.
For the proton, $q = +e$ and $\Delta V = -1$ V, so $KE_f = -(e)(-1\text{ V}) = +1$ eV, the same as that of the electron.

8. Choice (c). The battery moves negative charge from one plate and puts it on the other. The first plate is left with excess positive charge whose magnitude equals that of the negative charge moved to the other plate.

9. (a) C decreases. (b) Q stays the same. (c) E stays the same.
(d) ΔV increases. (e) The energy stored increases.

68719_16_ch16_p030-061.indd 30 1/7/11 2:32:36 PM

Because the capacitor is removed from the battery, charges on the plates have nowhere to go. Thus, the charge on the capacitor plates remains the same as the plates are pulled apart. Because $E = \sigma/\epsilon_0 = (Q/A)/\epsilon_0$, the electric field is constant as the plates are separated. Because $\Delta V = Ed$ and E does not change, ΔV increases as d increases. Because the same charge is stored at a higher potential difference, the capacitance ($C = Q/\Delta V$) has decreased. Because energy stored $= Q^2/2C$ and Q stays the same while C decreases, the energy stored increases. The extra energy must have been transferred from somewhere, so work was done. This is consistent with the fact that the plates attract one another, and work must be done to pull them apart.

10. (a) C increases. (b) Q increases. (c) E stays the same. (d) ΔV remains the same. (e) The energy stored increases.

The presence of a dielectric between the plates increases the capacitance by a factor equal to the dielectric constant. Since the battery holds the potential difference constant while the capacitance increases, the charge stored ($Q = C\Delta V$) will increase. Because the potential difference and the distance between the plates are both constant, the electric field ($E = \Delta V/d$) will stay the same. The battery maintains a constant potential difference. With ΔV constant while capacitance increases, the stored energy [energy stored $= \frac{1}{2}C(\Delta V)^2$] will increase.

11. Choice (a). Increased random motions associated with an increase in temperature make it more difficult to maintain a high degree of polarization of the dielectric material. This has the effect of decreasing the dielectric constant of the material, and in turn, decreasing the capacitance of the capacitor.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The change in the potential energy of the proton is equal to the negative of the work done on it by the electric field. Thus,

$$\Delta PE = -W = -qE_x(\Delta x) = -\left(+1.6\times10^{-19}\text{ C}\right)\left(850\text{ N/C}\right)\left(2.5\text{ m} - 0\right) = -3.4\times10^{-16}\text{ J}$$

and (b) is the correct choice for this question.

2. Because electric forces are conservative, the kinetic energy gained is equal to the decrease in electrical potential energy, or

$$KE = -PE = -q(\Delta V) = -(-1\text{ e})\left(+1.00\times10^{4}\text{ V}\right) = +1.00\times10^{4}\text{ eV}$$

so the correct choice is (a).

3. In a uniform electric field, the change in electric potential is $\Delta V = -E_x(\Delta x)$, giving

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{\left(V_f - V_i\right)}{\left(x_f - x_i\right)} = -\frac{(190\text{ V} - 120\text{ V})}{(5.0\text{ m} - 3.0\text{ m})} = -35\text{ V/m} = -35\text{ N/C}$$

and it is seen that the correct choice is (d).

4. From conservation of energy, $KE_f + PE_f = KE_i + PE_i$, or $\frac{1}{2}mv_B^2 = \frac{1}{2}mv_A^2 + qV_A - qV_B$, the final speed of the nucleus is

$$v_B = \sqrt{v_A^2 + \frac{2q(V_A - V_B)}{m}}$$

$$= \sqrt{\left(6.20\times10^5 \text{ m/s}\right)^2 + \frac{2\left[2\left(1.60\times10^{-19} \text{ C}\right)(1.50-4.00)\times10^3 \text{ V}\right]}{6.63\times10^{-27} \text{ kg}}} = 3.78\times10^5 \text{ m/s}$$

Thus, the correct answer is choice (b).

5. In a series combination of capacitors, the equivalent capacitance is always less than any individual capacitance in the combination, meaning that choice (a) is false. Also, for a series combination of capacitors, the magnitude of the charge is the same on all plates of capacitors in the combination, making both choices (d) and (e) false. The potential difference across the capacitance C_i is $\Delta V_i = Q/C_i$, where Q is the common charge on each capacitor in the combination. Thus, the largest potential difference (voltage) appears across the capacitor with the *least* capacitance, making choice (b) the correct answer.

6. The total potential at a point due to a set of point charges q_i is

$$V = \sum_i kq_i/r_i$$

where r_i is the distance from the point of interest to the location of the charge q_i. Note that in this case, the point at the center of the circle is equidistant from the 4 point charges located on the rim of the circle. Note also that $q_2 + q_3 + q_4 = (+1.5 - 1.0 - 0.5)\ \mu\text{C} = 0$, so we have

$$V_{\text{center}} = \frac{k_e q_1}{r} + \frac{k_e q_2}{r} + \frac{k_e q_3}{r} + \frac{k_e q_4}{r} = \frac{k_e}{r}\left(q_1 + q_2 + q_3 + q_4\right) = \frac{k_e}{r}\left(q_1 + 0\right) = \frac{k_e q_1}{r} = V_1$$

$$= 4.5\times10^4 \text{ V}$$

or the total potential at the center of the circle is just that due to the first charge alone, and the correct answer is choice (b).

7. With the given specifications, the capacitance of this parallel-plate capacitor will be

$$C = \frac{\kappa \in_0 A}{d} = \frac{\left(1.00\times10^2\right)\left(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(1.00 \text{ cm}^2\right)}{1.00\times10^{-3} \text{ m}}\left(\frac{1 \text{ m}^2}{10^4 \text{ cm}^2}\right)$$

$$= 8.85\times10^{-11} \text{ F} = 88.5\times10^{-12} \text{ F} = 88.5 \text{ pF}$$

and the correct choice is (a).

8. Keeping the capacitor connected to the battery means that the potential difference between the plates is kept at a constant value equal to the voltage of the battery. Since the capacitance of a parallel-plate capacitor is $C = \kappa \in_0 A/d$, doubling the plate separation d, while holding other characteristics of the capacitor constant, means the capacitance will be decreased by a factor of 2. The energy stored in a capacitor may be expressed as $U = \frac{1}{2}C(\Delta V)^2$, so when the potential difference ΔV is held constant while the capacitance is decreased by a factor of 2, the stored energy decreases by a factor of 2, making (c) the correct choice for this question.

9. When the battery is disconnected, there is no longer a path for charges to use in moving onto or off of the plates of the capacitor. This means that the charge Q is constant. The capacitance of a parallel-plate capacitor is $C = \kappa \in_0 A/d$ and the dielectric constant is $\kappa \approx 1$ when the capacitor is air filled. When a dielectric with dielectric constant $\kappa = 2$ is inserted between the plates, the capacitance is doubled $\left(C_f = 2C_i\right)$. Thus, with Q constant, the potential difference between the plates, $\Delta V = Q/C$, is decreased by a factor of 2, meaning that choice (a) is a true statement. The electric field between the plates of a parallel-plate capacitor is $E = \Delta V/d$ and decreases when ΔV decreases, making choice (e) false and leaving (a) as the only correct choice for this question.

10. Once the capacitor is disconnected from the battery, there is no path for charges to move onto or off of the plates, so the charges on the plates are constant, and choice (e) can be eliminated. The capacitance of a parallel-plate capacitor is $C = \kappa \in_0 A/d$, so the capacitance decreases when the plate separation d is increased. With Q constant and C decreasing, the energy stored in the capacitor, $U = Q^2/2C$, increases, making choice (a) false and choice (b) true. The potential difference between the plates, $\Delta V = Q/C = Q \cdot d/\kappa\in_0 A$, increases and the electric field between the plates, $E = \Delta V/d = Q/\kappa\in_0 A$, is constant. This means that both choices (c) and (d) are false and leaves choice (b) as the only correct response.

11. Capacitances connected in parallel all have the same potential difference across them and the equivalent capacitance, $C_{eq} = C_1 + C_2 + C_3 + \ldots$, is larger than the capacitance of any one of the capacitors in the combination. Thus, choice (c) is a true statement. The charge on a capacitor is $Q = C(\Delta V)$, so with ΔV constant, but the capacitances different, the capacitors all store different charges that are proportional to the capacitances, making choices (a), (b), (d), and (e) all false. Therefore, (c) is the only correct answer.

12. For a series combination of capacitors, the magnitude of the charge is the same on all plates of capacitors in the combination. Also, the equivalent capacitance is always less than any individual capacitance in the combination. Therefore, choice (a) is true while choices (b) and (c) are both false. The potential difference across a capacitor is $\Delta V = Q/C$, so with Q constant, capacitors having different capacitances will have different potential differences across them, with the largest potential difference being across the capacitor with the smallest capacitance. This means that choices (d) and (e) are false, and choice (f) is true. Thus, both choices (a) and (f) are true statements.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The potential energy between a pair of point charges separated by distance R is $PE = k_e\, q_1 q_2/R$. Thus, the potential energy for each of the four systems is:

(a) $PE_a = k_e \dfrac{Q(2Q)}{r} = 2k_e \dfrac{Q^2}{r}$ (b) $PE_b = k_e \dfrac{(-Q)(-Q)}{r} = k_e \dfrac{Q^2}{r}$

(c) $PE_c = k_e \dfrac{Q(-Q)}{2r} = -\dfrac{1}{2} k_e \dfrac{Q^2}{r}$ (d) $PE_d = k_e \dfrac{(-Q)(-2Q)}{2r} = k_e \dfrac{Q^2}{r}$

Therefore, the correct ranking from largest to smallest is $(a) > (b) = (d) > (c)$.

4. To move like charges together from an infinite separation, at which the potential energy of the system of two charges is zero, requires *work* to be done on the system by an outside agent. Hence energy is stored, and potential energy is positive. As charges with opposite signs move together from an infinite separation, energy is released, and the potential energy of the set of charges becomes negative.

6. A sharp point on a charged conductor would produce a large electric field in the region near the point. An electric discharge could most easily take place at the point.

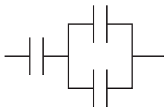
8. There are eight different combinations that use all three capacitors in the circuit. These combinations and their equivalent capacitances are:

All three capacitors in series: $C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$

All three capacitors in parallel: $C_{\text{eq}} = C_1 + C_2 + C_3$

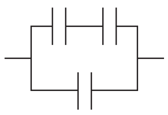
One capacitor in series with a parallel combination of the other two:

$$C_{\text{eq}} = \left(\frac{1}{C_1 + C_2} + \frac{1}{C_3}\right)^{-1},\ C_{\text{eq}} = \left(\frac{1}{C_3 + C_1} + \frac{1}{C_2}\right)^{-1},\ C_{\text{eq}} = \left(\frac{1}{C_2 + C_3} + \frac{1}{C_1}\right)^{-1}$$



One capacitor in parallel with a series combination of the other two:

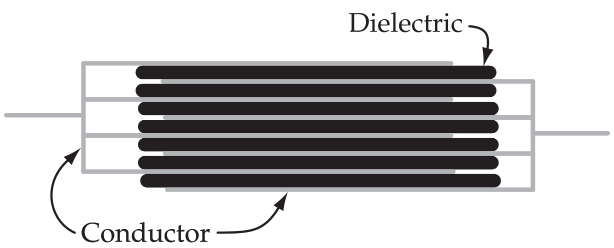
$$C_{\text{eq}} = \left(\frac{C_1C_2}{C_1 + C_2}\right) + C_3,\ C_{\text{eq}} = \left(\frac{C_3C_1}{C_3 + C_1}\right) + C_2,\ C_{\text{eq}} = \left(\frac{C_2C_3}{C_2 + C_3}\right) + C_1$$



10. (a) If the wires are disconnected from the battery and not allowed to touch each other or another object, the charge on the plates is unchanged.

(b) If, after being disconnected from the battery, the wires are connected to each other, electrons will rapidly flow from the negatively charged plate to the positively charged plate to leave the capacitor uncharged with both plates neutral.

12. The primary choice would be the dielectric. You would want to choose a dielectric that has a large dielectric constant and dielectric strength, such as strontium titanate, where $\kappa \approx 233$ (Table 16.1). A convenient choice could be thick plastic or Mylar. Secondly, geometry would be a factor. To maximize capacitance, one would want the individual plates as close as possible, since the capacitance is proportional to the inverse of the plate separation—hence the need for a dielectric with a high dielectric strength. Also, one would want to build, instead of a single parallel-plate capacitor, several capacitors in parallel. This could be achieved through "stacking" the plates of the capacitor. For example, you can alternately lay down sheets of a conducting material, such as aluminum foil, sandwiched between your sheets of insulating dielectric. Making sure that none of the conducting sheets are in contact with their nearest neighbors, connect every other plate together as illustrated in the figure below.



This technique is often used when "home-brewing" signal capacitors for radio applications, as they can withstand huge potential differences without flashover (without either discharge between plates around the dielectric or dielectric breakdown). One variation on this technique is to sandwich together flexible materials such as aluminum roof flashing and thick plastic, so the whole product can be rolled up into a "capacitor burrito" and placed in an insulating tube, such as a PVC pipe, and then filled with motor oil (again to prevent flashover).

68719_16_ch16_p030-061.indd 34 1/7/11 2:32:43 PM

14. The material of the dielectric may be able to withstand a larger electric field than air can withstand before breaking down to pass a spark between the capacitor plates.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 6.16×10^{-17} N

(b) 3.69×10^{10} m/s^2 in the direction of the electric field

(c) 7.38 cm

4. 6.67×10^{11} electrons

6. **(a)** 1.10×10^{-2} N to the right **(b)** 1.98×10^{-3} J **(c)** -1.98×10^{-3} J

(d) -49.5 V

8. **(a)** -2.31 kV **(b)** Protons would require a greater potential difference.

(c) $\Delta V_p / \Delta V_e = -m_p / m_e$

10. 40.2 kV

12. **(a)** +5.39 kV **(b)** +10.8 kV

14. -9.08 J

16. **(a)** See Solution. **(b)** $3k_e q/a$ **(c)** See Solution.

18. **(a)** See Solution. **(b)** $V = (22.5 \text{ V}\cdot\text{m})\left(\frac{1}{1.20 \text{ m} - x} - \frac{2}{x}\right)$

(c) -37.5 V **(d)** $x = 0.800$ m

20. **(a)** Conservation of energy alone yields one equation with two unknowns.

(b) Conservation of linear momentum

(c) $v_p = 1.05 \times 10^7$ m/s, $v_\alpha = 2.64 \times 10^6$ m/s

22. 5.4×10^5 V

24. **(a)** $V = 4\sqrt{2}k_e Q/a$ **(b)** $W = 4\sqrt{2}k_e qQ/a$

26. **(a)** 3.00 μF **(b)** 36.0 μC

28. **(a)** 1.00 μF **(b)** 100 V

30. 31.0 Å

32. 1.23 kV

34. **(a)** 17.0 μF **(b)** 9.00 V

(c) 45.0 μC on C_1, 108 μC on C_2

36. 3.00 pF and 6.00 pF

38. **(a)** 6.00 μF **(b)** 12.0 μF **(c)** 432 μC

(d) $Q_4 = 144\ \mu\text{C}$, $Q_2 = 72.0\ \mu\text{C}$, $Q_{\substack{\text{rightmost}\\\text{branch}}} = 216\ \mu\text{C}$ **(e)** $Q_{24} = Q_8 = 216\ \mu\text{C}$

(f) 9.00 V **(g)** 27.0 V

40. **(a)** $2C$ **(b)** $Q_1 > Q_3 > Q_2$ **(c)** $\Delta V_1 > \Delta V_2 = \Delta V_3$

(d) Q_1 and Q_3 increase, Q_2 decreases

42. **(a)** 6.04 μF **(b)** 83.6 μC

44. **(a)** 5.96 μF

(b) 89.4 μC on the 20.0 μF capacitor, 63.0 μC on the 6.00 μF capacitor, 26.3 μC on the 15.0 μF capacitor, and 26.3 μC on the 3.00 μF capacitor

46. **(a)** $C_{\text{eq}} = 12.0\ \mu\text{F}$, $E_{\text{stored,total}} = 8.64 \times 10^{-4}$ J

(b) $E_{\text{stored,1}} = 5.76 \times 10^{-4}$ J, $E_{\text{stored,2}} = 2.88 \times 10^{-4}$ J

It will always be true that $E_{\text{stored,1}} + E_{\text{stored,2}} = E_{\text{stored,total}}$.

(c) 5.66 V; C_2, with the largest capacitance, stores the most energy.

48. 9.79 kg

50. **(a)** 13.3 nC **(b)** 272 nC

52. 1.04 m

54. 0.443 mm

56. **(a)** 13.5 mJ

(b) $E_{\text{stored,2}} = 3.60$ mJ, $E_{\text{stored,3}} = 5.40$ mJ, $E_{\text{stored,4}} = 1.80$ mJ, $E_{\text{stored,6}} = 2.70$ mJ

(c) The energy stored in the equivalent capacitance equals the sum of the energies stored in the individual capacitors.

58. $C_1 = \frac{1}{2}C_p \pm \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$, $\quad C_2 = \frac{1}{2}C_p \mp \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$

60. **(a)** 1.8×10^4 V **(b)** -3.6×10^4 V **(c)** -1.8×10^4 V

(d) -5.4×10^{-2} J

62. **(a)** $C = \frac{ab}{k_e(b-a)}$ **(b)** See Solution.

64. $\kappa = 2.33$

66. **(a)** $\dfrac{2k_e q}{d\sqrt{5}}$ **(b)** $\dfrac{4k_e q^2}{d\sqrt{5}}$ **(c)** $\dfrac{4k_e q^2}{d\sqrt{5}}$

(d) $\sqrt{\dfrac{8k_e q^2}{md\sqrt{5}}}$

68. **(a)** 0.1 mm **(b)** 4.4 mm

PROBLEM SOLUTIONS

16.1 (a) Because the electron has a negative charge, it experiences a force in the direction opposite to the field and, when released from rest, will move in the negative x-direction. The work done on the electron by the field is

$$W = F_x(\Delta x) = (qE_x)\Delta x = (-1.60\times10^{-19}\ \text{C})(375\ \text{N/C})(-3.20\times10^{-2}\ \text{m})$$
$$= \boxed{1.92\times10^{-18}\ \text{J}}$$

(b) The change in the electric potential energy is the negative of the work done on the particle by the field. Thus,

$$\Delta PE = -W = \boxed{-1.92\times10^{-18}\ \text{J}}$$

(c) Since the Coulomb force is a conservative force, conservation of energy gives $\Delta KE + \Delta PE = 0$, or $KE_f = \frac{1}{2}m_e v_f^2 = KE_i - \Delta PE = 0 - \Delta PE$, and

$$v_f = \sqrt{\frac{-2(\Delta PE)}{m_e}} = \sqrt{\frac{-2(-1.92\times10^{-18}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}} = \boxed{2.05\times10^{6}\ \text{m/s in the } -x\text{-direction}}$$

16.2 (a) $F = qE = (1.60\times10^{-19}\ \text{C})(385\ \text{N/C}) = \boxed{6.16\times10^{-17}\ \text{N}}$

(b) $a = \dfrac{F}{m_p} = \dfrac{6.16\times10^{-17}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = \boxed{3.69\times10^{10}\ \text{m/s}^2 \text{ in the direction of the electric field}}$

(c) $\Delta x = v_0 t + \dfrac{1}{2}at^2 = 0 + \dfrac{1}{2}(3.69\times10^{10}\ \text{m/s}^2)(2.00\times10^{-6}\ \text{s})^2$

$= 7.38\times10^{-2}\ \text{m} = \boxed{7.38\ \text{cm}}$

16.3 The work done by the agent moving the charge out of the cell is

$$W_{\text{input}} = -W_{\text{field}} = -(-\Delta PE_e) = +q(\Delta V)$$
$$= (1.60\times10^{-19}\ \text{C})(+90\times10^{-3}\ \text{J/C}) = \boxed{1.4\times10^{-20}\ \text{J}}$$

16.4 Assuming the sphere is isolated, the excess charge on it is uniformly distributed over its surface. Under this spherical symmetry, the electric field outside the sphere is the same as if all the excess charge on the sphere were concentrated as a point charge located at the center of the sphere.

continued on next page

Thus, at $r = 8.00$ cm $> R_{\text{sphere}} = 5.00$ cm, the electric field is $E = k_e Q/r^2$. The required charge then has magnitude $|Q| = Er^2/k_e$, and the number of electrons needed is

$$n = \frac{|Q|}{e} = \frac{Er^2}{k_e e} = \frac{(1.50\times10^5\ \text{N/C})(8.00\times10^{-2}\ \text{m})^2}{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})} = \boxed{6.67\times10^{11}\ \text{electrons}}$$

16.5 $E = \dfrac{|\Delta V|}{d} = \dfrac{25\times10^3\ \text{J/C}}{1.5\times10^{-2}\ \text{m}} = \boxed{1.7\times10^6\ \text{N/C}}$

16.6 (a) $\vec{\mathbf{F}} = q\vec{\mathbf{E}} = (+40.0\times10^{-6}\ \text{C})(+275\ \text{N/C}) = \boxed{1.10\times10^{-2}\ \text{N directed toward the right}}$

(b) $W_{AB} = F(\Delta x)\cos\theta = (1.10\times10^{-2}\ \text{N})(0.180\ \text{m})\cos 0° = \boxed{1.98\times10^{-3}\ \text{J}}$

(c) $\Delta PE = -W_{AB} = \boxed{-1.98\times10^{-3}\ \text{J}}$

(d) $\Delta V = V_B - V_A = \dfrac{\Delta PE}{q} = \dfrac{-1.98\times10^{-3}\ \text{J}}{+40.0\times10^{-6}\ \text{C}} = \boxed{-49.5\ \text{V}}$

16.7 (a) $E = \dfrac{|\Delta V|}{d} = \dfrac{600\ \text{J/C}}{5.33\times10^{-3}\ \text{m}} = \boxed{1.13\times10^5\ \text{N/C}}$

(b) $F = |q|E = \dfrac{|q||\Delta V|}{d} = \dfrac{(1.60\times10^{-19}\ \text{C})(600\ \text{J/C})}{5.33\times10^{-3}\ \text{m}} = \boxed{1.80\times10^{-14}\ \text{N}}$

(c) $W = F\cdot s\cos\theta$

$$= (1.80\times10^{-14}\ \text{N})\left[(5.33-2.90)\times10^{-3}\ \text{m}\right]\cos 0° = \boxed{4.37\times10^{-17}\ \text{J}}$$

16.8 (a) Using conservation of energy, $\Delta KE + \Delta PE = 0$, with $KE_f = 0$ since the particle is "stopped," we have

$$\Delta PE = -\Delta KE = -\left(0 - \frac{1}{2}m_e v_i^2\right) = +\frac{1}{2}(9.11\times10^{-31}\ \text{kg})(2.85\times10^7\ \text{m/s})^2$$
$$= +3.70\times10^{-16}\ \text{J}$$

The required stopping potential is then

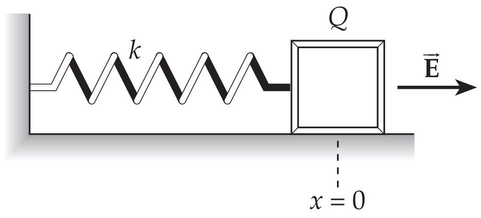
$$\Delta V = \frac{\Delta PE}{q} = \frac{+3.70\times10^{-16}\ \text{J}}{-1.60\times10^{-19}\ \text{C}} = -2.31\times10^3\ \text{V} = \boxed{-2.31\ \text{kV}}$$

(b) Being more massive than electrons, protons traveling at the same initial speed will have more initial kinetic energy and $\boxed{\text{require a greater magnitude stopping potential}}$.

(c) Since $\Delta V_{\text{stopping}} = \Delta PE/q = (-\Delta KE)/q = (-mv^2/2)/q$, the ratio of the stopping potential for a proton to that for an electron having the same initial speed is

$$\frac{\Delta V_p}{\Delta V_e} = \frac{-m_p v_i^2/2(+e)}{-m_e v_i^2/2(-e)} = \boxed{-m_p/m_e}$$

16.9 (a) We use conservation of energy, $\Delta(KE)+\Delta(PE_s)+\Delta(PE_e)=0$, recognizing that $\Delta(KE)=0$ since the block is at rest at both the beginning and end of the motion. The change in the elastic potential energy is given by $\Delta(PE_s)=\frac{1}{2}kx_{max}^2-0$, where x_{max} is the maximum stretch of the spring. The change in the electrical potential energy is the negative of the work the electric field does, $\Delta(PE_e)=-W=-F_e(\Delta x)=-(QE)x_{max}$. Thus, $0+\frac{1}{2}kx_{max}^2-(QE)x_{max}=0$, which yields



$$x_{max}=\frac{2QE}{k}=\frac{2(35.0\times10^{-6}\text{ C})(4.86\times10^{4}\text{ V/m})}{78.0\text{ N/m}}=4.36\times10^{-2}\text{ m}=\boxed{4.36\text{ cm}}$$

(b) At equilibrium, $\Sigma F=F_s+F_e=0$, or $-kx_{eq}+QE=0$. Therefore,

$$x_{eq}=\frac{QE}{k}=\frac{1}{2}x_{max}=\boxed{2.18\text{ cm}}$$

The amplitude is the distance from the equilibrium position to each of the turning points (at $x=0$ and $x=4.36$ cm), so $\boxed{A=2.18\text{ cm}=x_{max}/2}$.

(c) From conservation of energy, $\Delta(KE)+\Delta(PE_s)+\Delta(PE_e)=0$, we have $0+\frac{1}{2}kx_{max}^2+Q\Delta V=0$. Since $x_{max}=2A$, this gives

$$\Delta V=-\frac{kx_{max}^2}{2Q}=-\frac{k(2A)^2}{2Q}\qquad\text{or}\qquad\boxed{\Delta V=-\frac{2kA^2}{Q}}$$

16.10 Using $\Delta y=v_{0y}t+\frac{1}{2}a_yt^2$ for the full flight gives $0=v_{0y}t_f+\frac{1}{2}a_yt_f^2$, or $a_y=-2v_{0y}/t_f$, where t_f is the full time of the flight. Then, using $v_y^2=v_{0y}^2+2a_y(\Delta y)$ for the upward part of the flight gives

$$(\Delta y)_{max}=\frac{0-v_{0y}^2}{2a_y}=\frac{-v_{0y}^2}{2(-2v_{0y}/t_f)}=\frac{v_{0y}t_f}{4}=\frac{(20.1\text{ m/s})(4.10\text{ s})}{4}=20.6\text{ m}$$

From Newton's second law,

$$a_y=\frac{\Sigma F_y}{m}=\frac{-mg-qE}{m}=-\left(g+\frac{qE}{m}\right)$$

Equating this to the earlier result gives $a_y=-(g+qE/m)=-2v_{0y}/t_f$, so the electric field strength is

$$E=\left(\frac{m}{q}\right)\left[\frac{2v_{0y}}{t_f}-g\right]=\left(\frac{2.00\text{ kg}}{5.00\times10^{-6}\text{ C}}\right)\left[\frac{2(20.1\text{ m/s})}{4.10\text{ s}}-9.80\text{ m/s}^2\right]=1.95\times10^{3}\text{ N/C}$$

Thus, $(\Delta V)_{max}=(\Delta y_{max})E=(20.6\text{ m})(1.95\times10^{3}\text{ N/C})=4.02\times10^{4}\text{ V}=\boxed{40.2\text{ kV}}$

16.11 (a) $$V_A=\frac{k_eq}{r_A}=\frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\text{ C})}{0.250\times10^{-2}\text{ m}}=\boxed{-5.75\times10^{-7}\text{ V}}$$

(b) $$V_B=\frac{k_eq}{r_B}=\frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\text{ C})}{0.750\times10^{-2}\text{ m}}=\boxed{-1.92\times10^{-7}\text{ V}}$$

continued on next page

$$\Delta V = V_B - V_A = -1.92\times10^{-7}\text{ V} - \left(-5.75\times10^{-7}\text{ V}\right) = \boxed{+3.83\times10^{-7}\text{ V}}$$

(c) $\boxed{\text{No}}$. The original electron will be repelled by the negatively charged particle which suddenly appears at point A. Unless the electron is fixed in place, it will move in the opposite direction, away from points A and B, thereby lowering the potential difference between these points.

16.12 (a) $$V_A = \sum_i \frac{k_e q_i}{r_i} = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-15.0\times10^{-9}\text{ C}}{2.00\times10^{-2}\text{ m}} + \frac{27.0\times10^{-9}\text{ C}}{2.00\times10^{-2}\text{ m}}\right) = \boxed{+5.39\text{ kV}}$$

(b) $$V_B = \sum_i \frac{k_e q_i}{r_i} = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-15.0\times10^{-9}\text{ C}}{1.00\times10^{-2}\text{ m}} + \frac{27.0\times10^{-9}\text{ C}}{1.00\times10^{-2}\text{ m}}\right) = \boxed{+10.8\text{ kV}}$$

16.13 (a) Calling the 2.00 μC charge q_3,

$$V = \sum_i \frac{k_e q_i}{r_i} = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{\sqrt{r_1^2 + r_2^2}}\right)$$

$$= \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{8.00\times10^{-6}\text{ C}}{0.060\ 0\text{ m}} + \frac{4.00\times10^{-6}\text{ C}}{0.030\ 0\text{ m}} + \frac{2.00\times10^{-6}\text{ C}}{\sqrt{(0.060\ 0)^2 + (0.030\ 0)^2}\text{ m}}\right)$$

$$V = \boxed{2.67\times10^6\text{ V}}$$

(b) Replacing 2.00×10^{-6} C by -2.00×10^{-6} C in part (a) yields

$$V = \boxed{2.13\times10^6\text{ V}}$$

16.14 $W = q(\Delta V) = q\left(V_f - V_i\right)$, and $V_f = 0$ since the final location of the 8.00 μC is an infinite distance from other charges. The potential, due to the other charges, at the initial location of the 8.00 μC is $V_i = k_e\left(q_1/r_1 + q_2/r_2\right)$. Thus, the required energy for the move is

$$W = q\left[0 - k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)\right]$$

$$= -\left(8.00\times10^{-6}\text{ C}\right)\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.00\times10^{-6}\text{ C}}{0.030\ 0\text{ m}} + \frac{4.00\times10^{-6}\text{ C}}{\sqrt{(0.030\ 0)^2 + (0.060\ 0)^2}\text{ m}}\right)$$

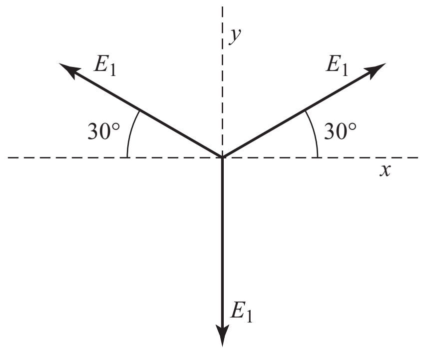
$$W = \boxed{-9.08\text{ J}}$$

16.15 (a) $$V = \sum_i \frac{k_e q_i}{r_i} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{5.00\times10^{-9}\text{ C}}{0.175\text{ m}} - \frac{3.00\times10^{-9}\text{ C}}{0.175\text{ m}}\right) = \boxed{103\text{ V}}$$

(b) $$PE = \frac{k_e q_1 q_2}{r_{12}} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(5.00\times10^{-9}\text{ C}\right)\left(-3.00\times10^{-9}\text{ C}\right)}{0.350\text{ m}} = \boxed{-3.85\times10^{-7}\text{ J}}$$

The negative sign means that $\boxed{\text{positive work must be done}}$ to separate the charges by an infinite distance (that is, bring them up to a state of zero potential energy).

16.16 (a) At the center of the triangle, each of the identical charges produce a field contribution of magnitude $E_1 = k_e q/a^2$. The three contributions are oriented as shown at the right and the components of the resultant field are:



$$E_x = \Sigma E_x = +E_1 \cos 30° - E_1 \cos 30° = 0$$

$$E_y = \Sigma E_y = +E_1 \sin 30° - E_1 + E_1 \sin 30° = 0$$

Thus, the resultant field has magnitude

$$E = \sqrt{E_x^2 + E_y^2} = \boxed{0}$$

(b) The total potential at the center of the triangle is

$$V = \Sigma V_i = \Sigma \frac{k_e q_i}{r_i} = \frac{k_e q}{a} + \frac{k_e q}{a} + \frac{k_e q}{a} = \boxed{\frac{3k_e q}{a}}$$

(c) Imagine a test charge placed at the center of the triangle. Since the field is zero at the center, the test charge will experience no electrical force at that point. The fact that the potential is not zero at the center means that work would have to be done by an external agent to move a test charge from infinity to the center.

16.17 The Pythagorean theorem gives the distance from the midpoint of the base to the charge at the apex of the triangle as

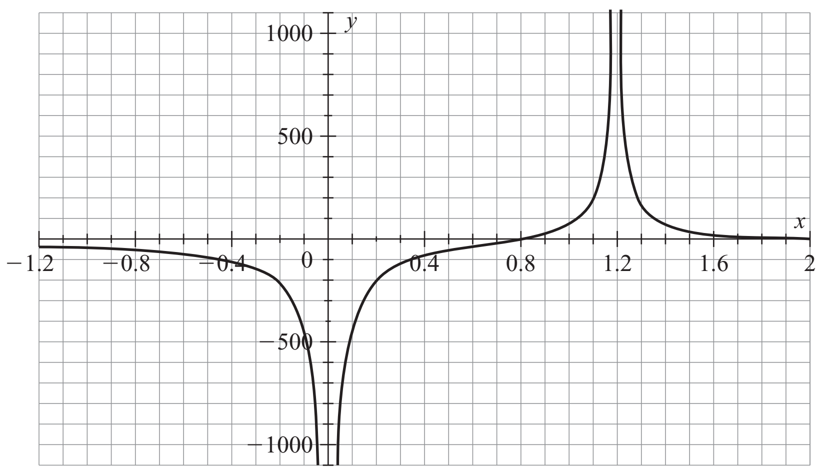
$$r_3 = \sqrt{(4.00 \text{ cm})^2 - (1.00 \text{ cm})^2} = \sqrt{15} \text{ cm} = \sqrt{15} \times 10^{-2} \text{ m}$$

Then, the potential at the midpoint of the base is $V = \sum_i k_e q_i / r_i$, or

$$V = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(\frac{(-7.00 \times 10^{-9} \text{ C})}{0.010\,0 \text{ m}} + \frac{(-7.00 \times 10^{-9} \text{ C})}{0.010\,0 \text{ m}} + \frac{(+7.00 \times 10^{-9} \text{ C})}{\sqrt{15} \times 10^{-2} \text{ m}}\right)$$

$$= -1.10 \times 10^4 \text{ V} = \boxed{-11.0 \text{ kV}}$$

16.18 (a) See the sketch below:



continued on next page

(b) At the point $(x, 0)$, where $0 < x < 1.20$ m, the potential is

$$V = \sum_i \frac{k_e q_i}{r_i} = \frac{k_e(-2q)}{x} + \frac{k_e q}{1.20\text{ m} - x} = k_e q\left(\frac{1}{1.20\text{ m} - x} - \frac{2}{x}\right)$$

or

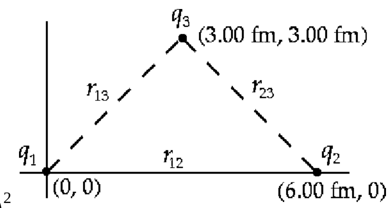
$$V = \left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(2.50\times10^{-9}\text{ C}\right)\left(\frac{1}{1.20\text{ m} - x} - \frac{2}{x}\right) = \boxed{(22.5\text{ V}\cdot\text{m})\left(\frac{1}{1.20\text{ m} - x} - \frac{2}{x}\right)}$$

(c) At $x = +0.600$ m, the potential is

$$V = (22.5\text{ V}\cdot\text{m})\left(\frac{1}{1.20\text{ m} - 0.600\text{ m}} - \frac{2}{0.600\text{ m}}\right) = -\frac{22.5\text{ V}\cdot\text{m}}{0.600\text{ m}} = \boxed{-37.5\text{ V}}$$

(d) When $0 < x < 1.20$ m and $V = 0$, we have $1/(1.20\text{ m} - x) - 2/x = 0$, or $x = 2.40\text{ m} - 2x$.

This yields $x = 2.40\text{ m}/3 = \boxed{0.800\text{ m}}$.



16.19 (a) When the charge configuration consists of only the two protons (q_1 and q_2 in the sketch), the potential energy of the configuration is

$$PE_a = \frac{k_e q_1 q_2}{r_{12}} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{6.00\times10^{-15}\text{ m}}$$

or $PE_a = \boxed{3.84\times10^{-14}\text{ J}}$

(b) When the alpha particle (q_3 in the sketch) is added to the configuration, there are three distinct pairs of particles, each of which possesses potential energy. The total potential energy of the configuration is now

$$PE_b = \frac{k_e q_1 q_2}{r_{12}} + \frac{k_e q_1 q_3}{r_{13}} + \frac{k_e q_2 q_3}{r_{23}} = PE_a + 2\left(\frac{k_e\left(2e^2\right)}{r_{13}}\right)$$

where use has been made of the facts that $q_1 q_3 = q_2 q_3 = e(2e) = 2e^2$ and

$r_{13} = r_{23} = \sqrt{(3.00\text{ fm})^2 + (3.00\text{ fm})^2} = \sqrt{18.0}\text{ fm} = \sqrt{18.0}\times10^{-15}$ m. Also, note that the first term in this computation is just the potential energy computed in part (a). Thus,

$$PE_b = PE_a + \frac{4k_e e^2}{r_{13}}$$

$$= 3.84\times10^{-14}\text{ J} + \frac{4\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\sqrt{18.0}\times10^{-15}\text{ m}} = \boxed{2.55\times10^{-13}\text{ J}}$$

(c) If we start with the three-particle system of part (b) and allow the alpha particle to escape to infinity [thereby returning us to the two-particle system of part (a)], the change in electric potential energy will be

$$\Delta PE = PE_a - PE_b = 3.84\times10^{-14}\text{ J} - 2.55\times10^{-13}\text{ J} = \boxed{-2.17\times10^{-13}\text{ J}}$$

(d) Conservation of energy, $\Delta KE + \Delta PE = 0$, gives the speed of the alpha particle at infinity in the situation of part (c) as $\frac{1}{2}m_\alpha v_\alpha^2 - 0 = -\Delta PE$, or

continued on next page

$$v_\alpha = \sqrt{\frac{-2(\Delta PE)}{m_\alpha}} = \sqrt{\frac{-2(-2.17\times10^{-13}\ \text{J})}{6.64\times10^{-27}\ \text{kg}}} = \boxed{8.08\times10^{6}\ \text{m/s}}$$

(e) When, starting with the three-particle system, the two protons are both allowed to escape to infinity, there will be no remaining pairs of particles and hence no remaining potential energy. Thus, $\Delta PE = 0 - PE_b = -PE_b$, and conservation of energy gives the change in kinetic energy as $\Delta KE = -\Delta PE = +PE_b$. Since the protons are identical particles, this increase in kinetic energy is split equally between them giving $KE_{\text{proton}} = \frac{1}{2}m_p v_p^2 = \frac{1}{2}(PE_b)$,

or $$v_p = \sqrt{\frac{PE_b}{m_p}} = \sqrt{\frac{2.55\times10^{-13}\ \text{J}}{1.67\times10^{-27}\ \text{kg}}} = \boxed{1.24\times10^{7}\ \text{m/s}}$$

16.20 (a) If a proton and an alpha particle, initially at rest 4.00 fm apart, are released and allowed to recede to infinity, the final speeds of the two particles will differ because of the difference in the masses of the particles. Thus, attempting to solve for the final speeds by use of conservation of energy alone leads to a situation of having [one equation with two unknowns], and does not permit a solution.

(b) In the situation described in part (a) above, one can obtain a second equation with the two unknown final speeds by using [conservation of linear momentum.] Then, one would have two equations which could be solved simultaneously for both unknowns.

(c) From conservation of energy: $\left[\left(\frac{1}{2}m_\alpha v_\alpha^2 + \frac{1}{2}m_p v_p^2\right) - 0\right] + \left[0 - k_e q_\alpha q_p / r_i\right] = 0$, or

$$m_\alpha v_\alpha^2 + m_p v_p^2 = \frac{2k_e q_\alpha q_p}{r_i} = \frac{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.20\times10^{-19}\ \text{C}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{4.00\times10^{-15}\ \text{m}}$$

yielding $m_\alpha v_\alpha^2 + m_p v_p^2 = 2.30\times10^{-13}\ \text{J}$ **[1]**

From conservation of linear momentum,

$$m_\alpha v_\alpha + m_p v_p = 0 \qquad \text{or} \qquad |v_\alpha| = \left(\frac{m_p}{m_\alpha}\right)v_p$$ **[2]**

Substituting Equation [2] into Equation [1] gives

$$m_\alpha\left(\frac{m_p}{m_\alpha}\right)^2 v_p^2 + m_p v_p^2 = 2.30\times10^{-13}\ \text{J} \qquad \text{or} \qquad \left(\frac{m_p}{m_\alpha}+1\right)m_p v_p^2 = 2.30\times10^{-13}\ \text{J}$$

and

$$v_p = \sqrt{\frac{2.30\times10^{-13}\ \text{J}}{\left(m_p/m_\alpha+1\right)m_p}} = \sqrt{\frac{2.30\times10^{-13}\ \text{J}}{\left(1.67\times10^{-27}/6.64\times10^{-27}+1\right)\left(1.67\times10^{-27}\ \text{kg}\right)}} = \boxed{1.05\times10^{7}\ \text{m/s}}$$

Then, Equation [2] gives the final speed of the alpha particle as

$$|v_\alpha| = \left(\frac{m_p}{m_\alpha}\right)v_p = \left(\frac{1.67\times10^{-27}\ \text{kg}}{6.64\times10^{-27}\ \text{kg}}\right)\left(1.05\times10^{7}\ \text{m/s}\right) = \boxed{2.64\times10^{6}\ \text{m/s}}$$

16.21 (a) Conservation of energy gives

$$KE_f = KE_i + \left(PE_i - PE_f\right) = 0 + k_e q_1 q_2 \left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

With $q_1 = +8.50$ nC, $q_2 = -2.80$ nC, $r_i = 1.60\ \mu$m, and $r_f = 0.500\ \mu$m, this becomes

$$KE_f = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(8.50\times10^{-9}\ \text{C}\right)\left(-2.80\times10^{-9}\ \text{C}\right)\left(\frac{1}{1.60\times10^{-6}\ \text{m}} - \frac{1}{0.500\times10^{-6}\ \text{m}}\right)$$

yielding $KE_f = \boxed{0.294\ \text{J}}$

(b) When $r = r_f = 0.500\ \mu$m and $KE = KE_f = 0.294$ J, the speed of the sphere having mass $m = 8.00\ \text{mg} = 8.00\times10^{-6}$ kg is

$$v_f = \sqrt{\frac{2\left(KE_f\right)}{m}} = \sqrt{\frac{2(0.294\ \text{J})}{8.00\times10^{-6}\ \text{kg}}} = \boxed{271\ \text{m/s}}$$

16.22 The excess charge on the metal sphere will be uniformly distributed over its surface. In this spherically symmetric situation, the electric field and the electric potential outside the sphere is the same as if all the excess charge were concentrated as a point charge at the center of the sphere. Thus, for points outside the sphere,

$$E = k_e\frac{Q}{r^2} \qquad \text{and} \qquad V = k_e\frac{Q}{r} = E\cdot r$$

If the sphere has a radius of $r = 18$ cm $= 0.18$ m and the air breaks down when $E = 3.0\times10^6$ V/m, the electric potential at the surface of the sphere when breakdown occurs is

$$V = \left(3.0\times10^6\ \text{V/m}\right)(0.18\ \text{m}) = \boxed{5.4\times10^5\ \text{V}}$$

16.23 From conservation of energy, $\left(KE + PE_e\right)_f = \left(KE + PE_e\right)_i$, which gives $0 + k_e Qq/r_f = \frac{1}{2}m_\alpha v_i^2 + 0$, or

$$r_f = \frac{2k_e Qq}{m_\alpha v_i^2} = \frac{2k_e(79e)(2e)}{m_\alpha v_i^2}$$

$$r_f = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(158)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(6.64\times10^{-27}\ \text{kg}\right)\left(2.00\times10^7\ \text{m/s}\right)^2} = \boxed{2.74\times10^{-14}\ \text{m}}$$

16.24 (a) The distance from any one of the corners of the square to the point at the center is one half the length of the diagonal of the square, or

$$r = \frac{diagonal}{2} = \frac{\sqrt{a^2 + a^2}}{2} = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$$

Since the charges have equal magnitudes and are all the same distance from the center of the square, they make equal contributions to the total potential. Thus,

$$V_{\text{total}} = 4V_{\substack{\text{single}\\\text{charge}}} = 4\frac{k_e Q}{r} = 4\frac{k_e Q}{a/\sqrt{2}} = \boxed{4\sqrt{2}k_e\frac{Q}{a}}$$

continued on next page

(b) The work required to carry charge q from infinity to the point at the center of the square is equal to the increase in the electric potential energy of the charge, or

$$W = PE_{\text{center}} - PE_{\infty} = qV_{\text{total}} - 0 = q\left(4\sqrt{2}k_e \frac{Q}{a}\right) = \boxed{4\sqrt{2}k_e \frac{qQ}{a}}$$

16.25 (a) $C = \epsilon_0 \dfrac{A}{d} = \left(8.85\times10^{-12}\ \dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\dfrac{\left(1.0\times10^{6}\ \text{m}^2\right)}{(800\ \text{m})} = \boxed{1.1\times10^{-8}\ \text{F}}$

(b) $Q_{\text{max}} = C(\Delta V)_{\text{max}} = C\left(E_{\text{max}} d\right) = \epsilon_0 \dfrac{A}{\cancel{d}}\left(E_{\text{max}} \cancel{d}\right) = \epsilon_0\ AE_{\text{max}}$

$$= \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.0\times10^{6}\ \text{m}^2\right)\left(3.0\times10^{6}\ \text{N/C}\right) = \boxed{27\ \text{C}}$$

16.26 (a) $C = \dfrac{Q}{\Delta V} = \dfrac{27.0\ \mu\text{C}}{9.00\ \text{V}} = \boxed{3.00\ \mu\text{F}}$

(b) $Q = C(\Delta V) = (3.00\ \mu\text{F})(12.0\ \text{V}) = \boxed{36.0\ \mu\text{C}}$

16.27 (a) The capacitance of this air-filled (dielectric constant, $\kappa = 1.00$) parallel-plate capacitor is

$$C = \frac{k \epsilon_0 A}{d} = \frac{(1.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.30\times10^{-4}\ \text{m}^2\right)}{1.50\times10^{-3}\ \text{m}} = 1.36\times10^{-12}\ \text{F} = \boxed{1.36\ \text{pF}}$$

(b) $Q = C(\Delta V) = \left(1.36\times10^{-12}\ \text{F}\right)(12.0\ \text{V}) = 1.63\times10^{-11}\ \text{C} = 16.3\times10^{-12}\ \text{C} = \boxed{16.3\ \text{pC}}$

(c) $E = \dfrac{\Delta V}{d} = \dfrac{12.0\ \text{V}}{1.50\times10^{-3}\ \text{m}} = \boxed{8.00\times10^{3}\ \text{V/m}} = \boxed{8.00\times10^{3}\ \text{N/C}}$

16.28 (a) $C = \dfrac{Q}{V} = \dfrac{10.0\ \mu\text{C}}{10.0\ \text{V}} = \boxed{1.00\ \mu\text{F}}$

(b) $V = \dfrac{Q}{C} = \dfrac{100\ \mu\text{C}}{1.00\ \mu\text{F}} = \boxed{100\ \text{V}}$

16.29 (a) $E = \dfrac{\Delta V}{d} = \dfrac{20.0\ \text{V}}{1.80\times10^{-3}\ \text{m}} = 1.11\times10^{4}\ \text{V/m} = \boxed{11.1\ \text{kV/m}}$ toward the negative plate

(b) $C = \dfrac{\epsilon_0 A}{d} = \dfrac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.60\times10^{-4}\ \text{m}^2\right)}{1.80\times10^{-3}\ \text{m}} = 3.74\times10^{-12}\ \text{F} = \boxed{3.74\ \text{pF}}$

(c) $Q = C(\Delta V) = \left(3.74\times10^{-12}\ \text{F}\right)(20.0\ \text{V}) = 7.48\times10^{-11}\ \text{C} = \boxed{74.8\ \text{pC}}$ on one plate and $\boxed{-74.8\ \text{pC}}$ on the other plate.

16.30 $C = \epsilon_0\ A/d$, so

$$d = \frac{\epsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(21.0\times10^{-12}\ \text{m}^2\right)}{60.0\times10^{-15}\ \text{F}} = 3.10\times10^{-9}\ \text{m}$$

$$d = \left(3.10\times10^{-9}\ \text{m}\right)\left(\frac{1\ \text{Å}}{10^{-10}\ \text{m}}\right) = \boxed{31.0\ \text{Å}}$$

16.31 (a) Assuming the capacitor is air-filled $(\kappa = 1)$, the capacitance is

$$C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.200\ \text{m}^2\right)}{3.00\times10^{-3}\ \text{m}} = \boxed{5.90\times10^{-10}\ \text{F}}$$

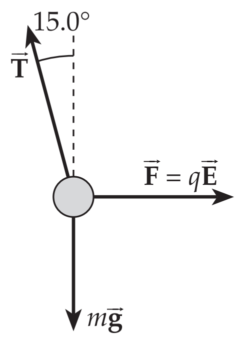
(b) $Q = C(\Delta V) = \left(5.90\times10^{-10}\ \text{F}\right)(6.00\ \text{V}) = \boxed{3.54\times10^{-9}\ \text{C}}$

(c) $E = \dfrac{\Delta V}{d} = \dfrac{6.00\ \text{V}}{3.00\times10^{-3}\ \text{m}} = \boxed{2.00\times10^{3}\ \text{V/m}} = \boxed{2.00\times10^{3}\ \text{N/C}}$

(d) $\sigma = \dfrac{Q}{A} = \dfrac{3.54\times10^{-9}\ \text{C}}{0.200\ \text{m}^2} = \boxed{1.77\times10^{-8}\ \text{C/m}^2}$

(e) Increasing the distance separating the plates decreases the capacitance, the charge stored, and the electric field strength between the plates. This means that all of the $\boxed{\text{previous answers will be decreased}}$.

16.32 $\Sigma F_y = 0 \ \Rightarrow\ T\cos 15.0° = mg$ or $T = \dfrac{mg}{\cos 15.0°}$



$\Sigma F_x = 0 \ \Rightarrow\ qE = T\sin 15.0° = mg\tan 15.0°$

or $E = \dfrac{mg\tan 15.0°}{q}$

$$\Delta V = Ed = \frac{mgd\tan 15.0°}{q}$$

$$\Delta V = \frac{\left(350\times10^{-6}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(0.040\ 0\ \text{m}\right)\tan 15.0°}{30.0\times10^{-9}\ \text{C}} = 1.23\times10^{3}\ \text{V} = \boxed{1.23\ \text{kV}}$$

16.33 (a) Capacitors in a series combination store the same charge, $Q = C_{eq}(\Delta V)$, where C_{eq} is the equivalent capacitance and ΔV is the potential difference maintained across the series combination. The equivalent capacitance for the given series combination is $1/C_{eq} = 1/C_1 + 1/C_2$, or $C_{eq} = C_1C_2/(C_1 + C_2)$, giving

$$C_{eq} = \frac{(2.50\ \mu\text{F})(6.25\ \mu\text{F})}{2.50\ \mu\text{F} + 6.25\ \mu\text{F}} = 1.79\ \mu\text{F}$$

and the charge stored on each capacitor in the series combination is

$$Q = C_{eq}(\Delta V) = (1.79\ \mu\text{F})(6.00\ \text{V}) = \boxed{10.7\ \mu\text{C}}$$

(b) When connected in parallel, each capacitor has the same potential difference, $\Delta V = 6.00$ V, maintained across it. The charge stored on each capacitor is then

For $C_1 = 2.50\ \mu$F: $Q_1 = C_1(\Delta V) = (2.50\ \mu\text{F})(6.00\ \text{V}) = \boxed{15.0\ \mu\text{C}}$

For $C_2 = 6.25\ \mu$F: $Q_2 = C_2(\Delta V) = (6.25\ \mu\text{F})(6.00\ \text{V}) = \boxed{37.5\ \mu\text{C}}$

68719_16_ch16_p030-061.indd 46 1/7/11 2:33:10 PM

16.34 (a) $C_{eq} = C_1 + C_2 = 5.00\ \mu F + 12.0\ \mu F = \boxed{17.0\ \mu F}$

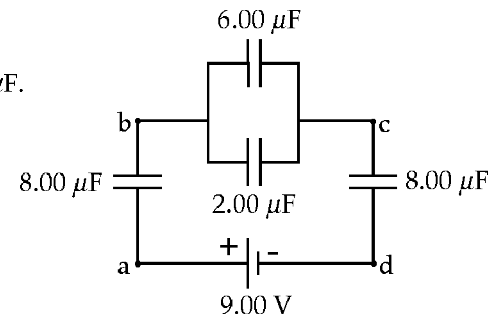
(b) In a parallel combination, the full potential difference maintained between the terminals of the battery exists across each capacitor. Thus,

$$\Delta V_1 = \Delta V_2 = \Delta V_{battery} = \boxed{9.00\ V}$$

(c) $Q_1 = C_1(\Delta V_1) = (5.00\ \mu F)(9.00\ V) = \boxed{45.0\ \mu C}$

$Q_2 = C_2(\Delta V_2) = (12.0\ \mu F)(9.00\ V) = \boxed{108\ \mu C}$

16.35 (a) First, we replace the parallel combination between points b and c by its equivalent capacitance, $C_{bc} = 2.00\ \mu F + 6.00\ \mu F = 8.00\ \mu F$. Then, we have three capacitors in series between points a and d. The equivalent capacitance for this circuit is therefore



$$\frac{1}{C_{eq}} = \frac{1}{C_{ab}} + \frac{1}{C_{bc}} + \frac{1}{C_{cd}} = \frac{3}{8.00\ \mu F}$$

giving $C_{eq} = \dfrac{8.00\ \mu F}{3} = \boxed{2.67\ \mu F}$

(b) The charge stored on each capacitor in the series combination is

$$Q_{ab} = Q_{bc} = Q_{cd} = C_{eq}(\Delta V_{ad}) = (2.67\ \mu F)(9.00\ V) = 24.0\ \mu C$$

Then, note that $\Delta V_{bc} = Q_{bc}/C_{bc} = 24.0\ \mu C/8.00\ \mu F = 3.00\ V$. The charge on each capacitor in the original circuit is:

On the $8.00\ \mu F$ between a and b: $Q_8 = Q_{ab} = \boxed{24.0\ \mu C}$

On the $8.00\ \mu F$ between c and d: $Q_8 = Q_{cd} = \boxed{24.0\ \mu C}$

On the $2.00\ \mu F$ between b and c: $Q_2 = C_2(\Delta V_{bc}) = (2.00\ \mu F)(3.00\ V) = \boxed{6.00\ \mu C}$

On the $6.00\ \mu F$ between b and c: $Q_6 = C_6(\Delta V_{bc}) = (6.00\ \mu F)(3.00\ V) = \boxed{18.0\ \mu C}$

(c) Note that $\Delta V_{ab} = Q_{ab}/C_{ab} = 24.0\ \mu C/8.00\ \mu F = 3.00\ V$, and that $\Delta V_{cd} = Q_{cd}/C_{cd} = 24.0\ \mu C/8.00\ \mu F = 3.00\ V$. We earlier found that $\Delta V_{bc} = 3.00\ V$, so we conclude that the potential difference across each capacitor in the circuit is

$$\Delta V_8 = \Delta V_2 = \Delta V_6 = \Delta V_8 = \boxed{3.00\ V}$$

16.36 $C_{parallel} = C_1 + C_2 = 9.00\ pF \quad \Rightarrow \quad C_1 = 9.00\ pF - C_2$ **[1]**

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \Rightarrow \quad C_{series} = \frac{C_1 C_2}{C_1 + C_2} = 2.00\ pF$$

Thus, using Equation [1], $C_{series} = \dfrac{(9.00\ pF - C_2)C_2}{(9.00\ pF - C_2) + C_2} = 2.00\ pF$ which reduces to

continued on next page

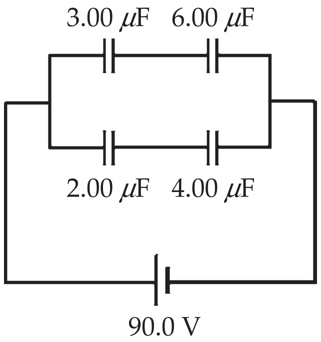
$$C_2^2 - (9.00\text{ pF})C_2 + 18.0\ (\text{pF})^2 = 0\text{, or }(C_2 - 6.00\text{ pF})(C_2 - 3.00\text{ pF}) = 0$$

Therefore, either $C_2 = 6.00$ pF and, from Equation [1], $C_1 = 3.00$ pF

or $C_2 = 3.00$ pF and $C_1 = 6.00$ pF.

We conclude that the two capacitances are $\boxed{3.00\text{ pF and }6.00\text{ pF}}$.

16.37 (a) The equivalent capacitance of the series combination in the upper branch is



$$\frac{1}{C_{\text{upper}}} = \frac{1}{3.00\ \mu\text{F}} + \frac{1}{6.00\ \mu\text{F}} = \frac{2+1}{6.00\ \mu\text{F}}$$

or $C_{\text{upper}} = 2.00\ \mu\text{F}$

Likewise, the equivalent capacitance of the series combination in the lower branch is

$$\frac{1}{C_{\text{lower}}} = \frac{1}{2.00\ \mu\text{F}} + \frac{1}{4.00\ \mu\text{F}} = \frac{2+1}{4.00\ \mu\text{F}} \quad\text{or}\quad C_{\text{lower}} = 1.33\ \mu\text{F}$$

These two equivalent capacitances are connected in parallel with each other, so the equivalent capacitance for the entire circuit is

$$C_{\text{eq}} = C_{\text{upper}} + C_{\text{lower}} = 2.00\ \mu\text{F} + 1.33\ \mu\text{F} = \boxed{3.33\ \mu\text{F}}$$

(b) Note that the same potential difference, equal to the potential difference of the battery, exists across both the upper and lower branches. The charge stored on each capacitor in the series combination in the upper branch is

$$Q_3 = Q_6 = Q_{\text{upper}} = C_{\text{upper}}(\Delta V) = (2.00\ \mu\text{F})(90.0\text{ V}) = \boxed{180\ \mu\text{C}}$$

and the charge stored on each capacitor in the series combination in the lower branch is

$$Q_2 = Q_4 = Q_{\text{lower}} = C_{\text{lower}}(\Delta V) = (1.33\ \mu\text{F})(90.0\text{ V}) = \boxed{120\ \mu\text{C}}$$

(c) The potential difference across each of the capacitors in the circuit is:

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\text{ V}} \qquad \Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\text{ V}}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\text{ V}} \qquad \Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\text{ V}}$$

16.38 (a) The equivalent capacitance of the series combination in the rightmost branch of the circuit is

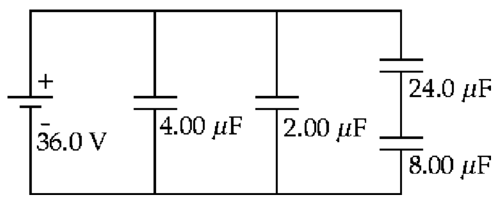


Figure P16.38

$$\frac{1}{C_{\text{right}}} = \frac{1}{24.0\ \mu\text{F}} + \frac{1}{8.00\ \mu\text{F}} = \frac{1+3}{24.0\ \mu\text{F}}$$

or $C_{\text{right}} = \boxed{6.00\ \mu\text{F}}$

continued on next page

(b) The equivalent capacitance of the three capacitors now connected in parallel with each other and with the battery is

$$C_{eq} = 4.00\ \mu\text{F} + 2.00\ \mu\text{F} + 6.00\ \mu\text{F} = \boxed{12.0\ \mu\text{F}}$$

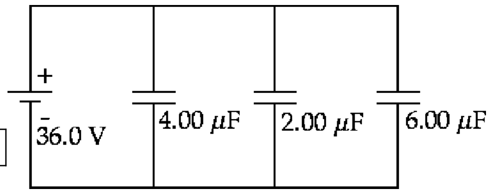


Diagram 1

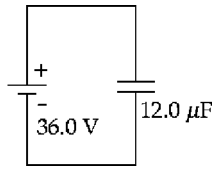


Diagram 2

(c) The total charge stored in this circuit is

$$Q_{total} = C_{eq}(\Delta V) = (12.0\ \mu\text{F})(36.0\ \text{V})$$

or $\quad Q_{total} = \boxed{432\ \mu\text{C}}$

(d) The charges on the three capacitors shown in Diagram 1 are:

$$Q_4 = C_4(\Delta V) = (4.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{144\ \mu\text{C}}$$

$$Q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{72\ \mu\text{C}}$$

$$Q_{right} = C_{right}(\Delta V) = (6.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{216\ \mu\text{C}}$$

$\boxed{\text{Yes.}}$ $Q_4 + Q_2 + Q_{right} = Q_{total}$ as it should.

(e) The charge on each capacitor in the series combination in the rightmost branch of the original circuit (Figure P16.38) is

$$Q_{24} = Q_8 = Q_{right} = \boxed{216\ \mu\text{C}}$$

(f) $$\Delta V_{24} = \frac{Q_{24}}{C_{24}} = \frac{216\ \mu\text{C}}{24.0\ \mu\text{F}} = \boxed{9.00\ \text{V}}$$

(g) $$\Delta V_8 = \frac{Q_8}{C_8} = \frac{216\ \mu\text{C}}{8.00\ \mu\text{F}} = \boxed{27.0\ \text{V}}$$ Note that $\Delta V_8 + \Delta V_{24} = \Delta V = 36.0\ \text{V}$ as it should.

16.39

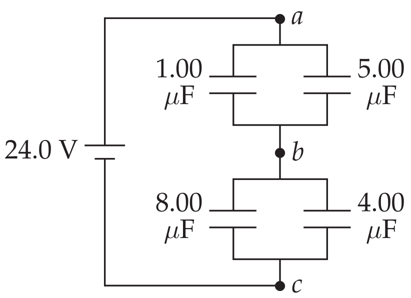


Figure 1

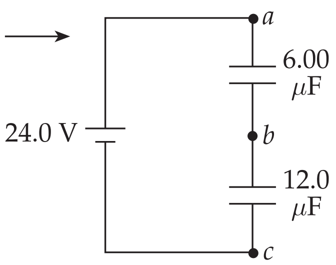


Figure 2

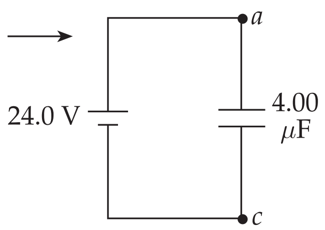


Figure 3

The circuit may be reduced in steps as shown above.

Using Figure 3, $\quad Q_{ac} = (4.00\ \mu\text{F})(24.0\ \text{V}) = 96.0\ \mu\text{C}$

Then, in Figure 2, $(\Delta V)_{ab} = \dfrac{Q_{ac}}{C_{ab}} = \dfrac{96.0\ \mu\text{C}}{6.00\ \mu\text{F}} = 16.0\ \text{V}$

continued on next page

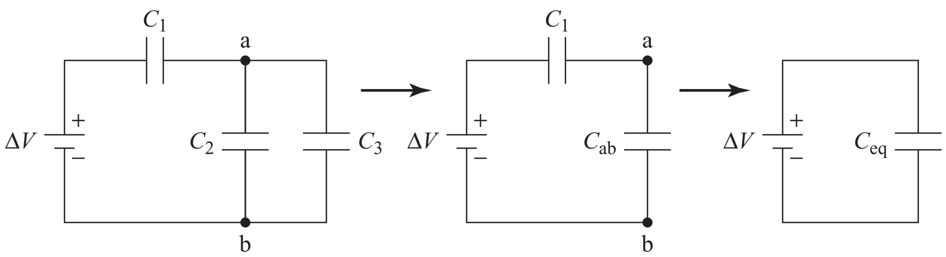
and $(\Delta V)_{bc} = (\Delta V)_{ac} - (\Delta V)_{ab} = 24.0\text{ V} - 16.0\text{ V} = 8.00\text{ V}$

Finally, using Figure 1,

$$Q_1 = C_1(\Delta V)_{ab} = (1.00\ \mu\text{F})(16.0\text{ V}) = \boxed{16.0\ \mu\text{C}},\quad Q_5 = (5.00\ \mu\text{F})(\Delta V)_{ab} = \boxed{80.0\ \mu\text{C}}$$

$$Q_8 = (8.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{64.0\ \mu\text{C}},\quad \text{and}\quad Q_4 = (4.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{32.0\ \mu\text{C}}$$

16.40 (a) Consider the simplification of the circuit as shown below:



Since C_2 and C_3 are connected in parallel, $C_{ab} = C_2 + C_3 = C + 5C = 6C$.

Now observe that C_1 and C_{ab} are connected in series, giving

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{ab}} \quad \text{or} \quad C_{eq} = \frac{C_1 C_{ab}}{C_1 + C_{ab}} = \frac{(3C)(6C)}{3C + 6C} = \boxed{2C}$$

(b) Since capacitors C_1 and C_{ab} are connected in series,

$$Q_1 = Q_{ab} = Q_{eq} = C_{eq}(\Delta V) = 2C(\Delta V)$$

Then, $\Delta V_{ab} = \dfrac{Q_{ab}}{C_{ab}} = \dfrac{2C(\Delta V)}{6C} = \dfrac{\Delta V}{3}$, giving $Q_2 = C_2(\Delta V_{ab}) = \dfrac{C(\Delta V)}{3}$

Also, $Q_3 = C_3(\Delta V_{ab}) = \dfrac{5C(\Delta V)}{3}$. Therefore, $\boxed{Q_1 > Q_3 > Q_2}$

(c) Since capacitors C_1 and C_{ab} are in series with the battery,

$$\Delta V_1 = \Delta V - \Delta V_{ab} = \Delta V - \frac{\Delta V}{3} = \frac{2}{3}\Delta V$$

Also, with capacitors C_2 and C_3 in parallel between points a and b,

$$\Delta V_2 = \Delta V_3 = \Delta V_{ab} = \frac{\Delta V}{3}$$

Thus, $\boxed{\Delta V_1 > \Delta V_2 = \Delta V_3}$

(d) Consider the following steps:

(i) Increasing C_3 while C_1 and C_2 remain constant will increase $C_{ab} = C_2 + C_3$.

Therefore, the equivalent capacitance, $C_{eq} = C_1\left(\dfrac{C_{ab}}{C_1 + C_{ab}}\right)$, will increase.

continued on next page

(ii) Since C_1 and C_{ab} are in series, $Q_1 = Q_{ab} = C_{eq}(\Delta V)$. Thus, $\boxed{Q_1 \text{ will increase}}$ as C_{eq} increases. Also, Q_{ab} experiences the same increase.

(iii) Because $\Delta V_1 = Q_1/C_1$, an increase in Q_1 causes ΔV_1 to increase and causes $\Delta V_{ab} = \Delta V - \Delta V_1$ to decrease. Thus, since $Q_2 = C_2(\Delta V_{ab})$, $\boxed{Q_2 \text{ will decrease}}$.

(iv) With capacitors C_2 and C_3 in parallel between points a and b, we have $Q_{ab} = Q_2 + Q_3$ or $Q_3 = Q_{ab} - Q_2$. Thus, with Q_{ab} increasing [see Step (ii)] while Q_2 is decreasing [see Step (iii)], we see that $\boxed{Q_3 \text{ will increase}}$.

16.41 (a) From $Q = C(\Delta V)$, $Q_{25} = (25.0\ \mu\text{F})(50.0\ \text{V}) = 1.25\times10^3\ \mu\text{C} = \boxed{1.25\ \text{mC}}$

and $Q_{40} = (40.0\ \mu\text{F})(50.0\ \text{V}) = 2.00\times10^3\ \mu\text{C} = \boxed{2.00\ \text{mC}}$

(b) Since the negative plate of one capacitor was connected to the positive plate of the other, the net charge stored in the new parallel combination is

$$Q = Q_{40} - Q_{25} = 2.00\times10^3\ \mu\text{C} - 1.25\times10^3\ \mu\text{C} = 750\ \mu\text{C}$$

The two capacitors, now in parallel, have a common potential difference ΔV across them. The new charges on each of the capacitors are $Q'_{25} = C_1(\Delta V)$ and $Q'_{40} = C_2(\Delta V)$. Thus,

$$Q'_{25} = \frac{C_1}{C_2}Q'_{40} = \left(\frac{25\ \mu\text{F}}{40\ \mu\text{F}}\right)Q'_{40} = \frac{5}{8}Q'_{40}$$

and the total change now stored in the combination may be written as

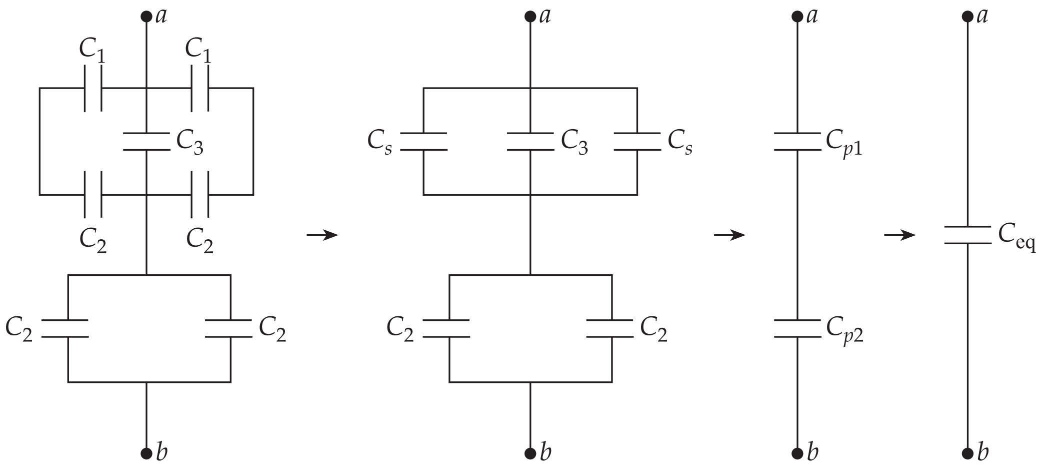
$$Q = Q'_{40} + Q'_{25} = Q'_{40} + \frac{5}{8}Q'_{40} = \frac{13}{8}Q'_{40} = 750\ \mu\text{C}$$

giving $Q'_{40} = \frac{8}{13}(750\ \mu\text{C}) = \boxed{462\ \mu\text{C}}$ and $Q'_{25} = Q - Q'_{40} = (750 - 462)\ \mu\text{C} = \boxed{288\ \mu\text{C}}$

(c) The potential difference across each capacitor in the new parallel combination is

$$\Delta V = \frac{Q}{C_{eq}} = \frac{Q}{C_1 + C_2} = \frac{750\ \mu\text{C}}{65.0\ \mu\text{F}} = \boxed{11.5\ \text{V}}$$

16.42 (a) The original circuit reduces to a single equivalent capacitor in the steps shown below.



continued on next page

$$C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{5.00\ \mu\text{F}} + \frac{1}{10.0\ \mu\text{F}}\right)^{-1} = 3.33\ \mu\text{F}$$

$$C_{p1} = C_s + C_3 + C_s = 2(3.33\ \mu\text{F}) + 2.00\ \mu\text{F} = 8.66\ \mu\text{F}$$

$$C_{p2} = C_2 + C_2 = 2(10.0\ \mu\text{F}) = 20.0\ \mu\text{F}$$

$$C_{\text{eq}} = \left(\frac{1}{C_{p1}} + \frac{1}{C_{p2}}\right)^{-1} = \left(\frac{1}{8.66\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$$

(b) The total charge stored between points *a* and *b* is

$$Q_{\text{total}} = C_{\text{eq}}(\Delta V)_{ab} = (6.04\ \mu\text{F})(60.0\ \text{V}) = 362\ \mu\text{C}$$

Then, looking at the third figure, observe that the charges of the series capacitors of that figure are $Q_{p1} = Q_{p2} = Q_{\text{total}} = 362\ \mu\text{C}$. Thus, the potential difference across the upper parallel combination shown in the second figure is

$$(\Delta V)_{p1} = \frac{Q_{p1}}{C_{p1}} = \frac{362\ \mu\text{C}}{8.66\ \mu\text{F}} = 41.8\ \text{V}$$

Finally, the charge on C_3 is

$$Q_3 = C_3(\Delta V)_{p1} = (2.00\ \mu\text{F})(41.8\ \text{V}) = \boxed{83.6\ \mu\text{C}}$$

16.43 From $Q = C(\Delta V)$, the initial charge of each capacitor is

$$Q_1 = (1.00\ \mu\text{F})(10.0\ \text{V}) = 10.0\ \mu\text{C} \quad \text{and} \quad Q_2 = (2.00\ \mu\text{F})(0) = 0$$

After the capacitors are connected in parallel, the potential difference across one is the same as that across the other. This gives

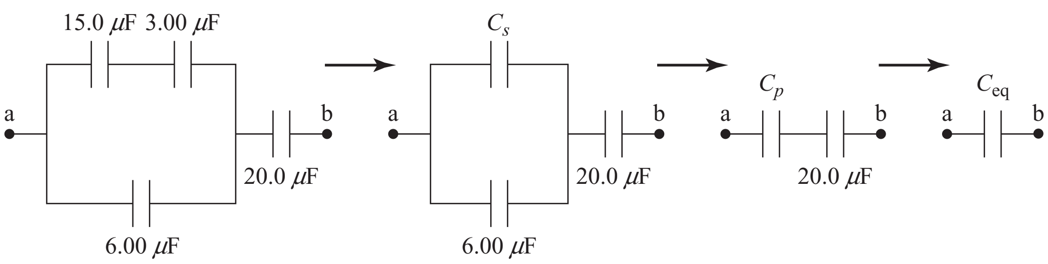
$$\Delta V = \frac{Q_1'}{1.00\ \mu\text{F}} = \frac{Q_2'}{2.00\ \mu\text{F}} \quad \text{or} \quad Q_2' = 2Q_1' \qquad \textbf{[1]}$$

From conservation of charge, $Q_1' + Q_2' = Q_1 + Q_2 = 10.0\ \mu\text{C}$. Then, substituting from Equation [1], this becomes

$$Q_1' + 2Q_1' = 10.0\ \mu\text{C}\text{, giving} \qquad Q_1' = \boxed{10\ \mu\text{C}/3} = \boxed{3.33\ \mu\text{C}}$$

Finally, from Equation [1], $\qquad Q_2' = \boxed{20\ \mu\text{C}/3} = \boxed{6.67\ \mu\text{C}}$

16.44 (a) We simplify the circuit in stages as shown below:



continued on next page

$$\frac{1}{C_s} = \frac{1}{15.0\ \mu\text{F}} + \frac{1}{3.00\ \mu\text{F}} \quad \text{or} \quad C_s = \frac{(15.0\ \mu\text{F})(3.00\ \mu\text{F})}{15.0\ \mu\text{F} + 3.00\ \mu\text{F}} = 2.50\ \mu\text{F}$$

$$C_p = C_s + 6.00\ \mu\text{F} = 2.50\ \mu\text{F} + 6.00\ \mu\text{F} = 8.50\ \mu\text{F}$$

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_p} + \frac{1}{20.0\ \mu\text{F}} \quad \text{or} \quad C_{\text{eq}} = \frac{(8.50\ \mu\text{F})(20.0\ \mu\text{F})}{8.50\ \mu\text{F} + 20.0\ \mu\text{F}} = \boxed{5.96\ \mu\text{F}}$$

(b) $Q_{20} = Q_{C_p} = Q_{\text{eq}} = C_{\text{eq}}(\Delta V_{\text{ab}}) = (5.96\ \mu\text{F})(15.0\ \text{V}) = \boxed{89.4\ \mu\text{C}}$

$$\Delta V_{C_p} = \frac{Q_{C_p}}{C_p} = \frac{89.4\ \mu\text{C}}{8.50\ \mu\text{F}} = 10.5\ \text{V}$$

so $Q_6 = C_6(\Delta V_{C_p}) = (6.00\ \mu\text{F})(10.5\ \text{V}) = \boxed{63.0\ \mu\text{C}}$

and $Q_{15} = Q_3 = Q_s = C_s(\Delta V_{C_p}) = (2.50\ \mu\text{F})(10.5\ \text{V}) = \boxed{26.3\ \mu\text{C}}$

The charges are 89.4 μC on the 20 μF capacitor, 63.0 μC on the 6 μF capacitor, and 26.3 μC on both the 15 μF and 3 μF capacitors.

16.45 Energy stored $= \dfrac{Q^2}{2C} = \dfrac{1}{2}C(\Delta V)^2 = \dfrac{1}{2}(4.50\times10^{-6}\ \text{F})(12.0\ \text{V})^2 = \boxed{3.24\times10^{-4}\ \text{J}}$

16.46 (a) The equivalent capacitance of a series combination of C_1 and C_2 is

$$\frac{1}{C_{\text{eq}}} = \frac{1}{18.0\ \mu\text{F}} + \frac{1}{36.0\ \mu\text{F}} = \frac{2+1}{36.0\ \mu\text{F}} \quad \text{or} \quad \boxed{C_{\text{eq}} = 12.0\ \mu\text{F}}$$

When this series combination is connected to a 12.0-V battery, the total stored energy is

$$\text{Total energy stored} = \frac{1}{2}C_{\text{eq}}(\Delta V)^2 = \frac{1}{2}(12.0\times10^{-6}\ \text{F})(12.0\ \text{V})^2 = \boxed{8.64\times10^{-4}\ \text{J}}$$

(b) The charge stored on each of the two capacitors in the series combination is

$$Q_1 = Q_2 = Q_{\text{total}} = C_{\text{eq}}(\Delta V) = (12.0\ \mu\text{F})(12.0\ \text{V}) = 144\ \mu\text{C} = 1.44\times10^{-4}\ \text{C}$$

and the energy stored in each of the individual capacitors is

$$\text{Energy stored in } C_1 = \frac{Q_1^2}{2C_1} = \frac{(1.44\times10^{-4}\ \text{C})^2}{2(18.0\times10^{-6}\ \text{F})} = \boxed{5.76\times10^{-4}\ \text{J}}$$

and $$\text{Energy stored in } C_2 = \frac{Q_2^2}{2C_2} = \frac{(1.44\times10^{-4}\ \text{C})^2}{2(36.0\times10^{-6}\ \text{F})} = \boxed{2.88\times10^{-4}\ \text{J}}$$

Energy stored in C_1 + Energy stored in $C_2 = 5.76\times10^{-4}\ \text{J} + 2.88\times10^{-4}\ \text{J} = 8.64\times10^{-4}\ \text{J}$, which is the same as the total stored energy found in part (a). This must be true

$\boxed{\text{if the computed equivalent capacitance is truly equivalent to the original combination.}}$

continued on next page

(c) If C_1 and C_2 had been connected in parallel rather than in series, the equivalent capacitance would have been $C_{eq} = C_1 + C_2 = 18.0\ \mu\text{F} + 36.0\ \mu\text{F} = 54.0\ \mu\text{F}$. If the total energy stored $\left[\frac{1}{2}C_{eq}(\Delta V)^2\right]$ in this parallel combination is to be the same as was stored in the original series combination, it is necessary that

$$\Delta V = \sqrt{\frac{2(\text{Total energy stored})}{C_{eq}}} = \sqrt{\frac{2(8.64\times10^{-4}\ \text{J})}{54.0\times10^{-6}\ \text{F}}} = \boxed{5.66\ \text{V}}$$

Since the two capacitors in parallel have the same potential difference across them, the energy stored in the individual capacitors $\left[\frac{1}{2}C(\Delta V)^2\right]$ is directly proportional to their capacitances. $\boxed{\text{The larger capacitor, } C_2\text{, stores the most energy in this case.}}$

16.47 (a) The energy initially stored in the capacitor is

$$(\text{Energy stored})_1 = \frac{Q_i^2}{2C_i} = \frac{1}{2}C_i(\Delta V)_i^2 = \frac{1}{2}(3.00\ \mu\text{F})(6.00\ \text{V})^2 = \boxed{54.0\ \mu\text{J}}$$

(b) When the capacitor is disconnected from the battery, the stored charge becomes isolated with no way off the plates. Thus, the charge remains constant at the value Q_i as long as the capacitor remains disconnected. Since the capacitance of a parallel-plate capacitor is $C = \kappa \epsilon_0 A/d$, when the distance d separating the plates is doubled, the capacitance is decreased by a factor of 2 $\left(C_f = C_i/2 = 1.50\ \mu\text{F}\right)$. The stored energy (with Q unchanged) becomes

$$(\text{Energy stored})_2 = \frac{Q_i^2}{2C_f} = \frac{Q_i^2}{2(C_i/2)} = 2\left(\frac{Q_i^2}{2C_i}\right) = 2(\text{Energy stored})_1 = \boxed{108\ \mu\text{J}}$$

(c) When the capacitor is reconnected to the battery, the potential difference between the plates is reestablished at the original value of $\Delta V = (\Delta V)_i = 6.00$ V, while the capacitance remains at $C_f = C_i/2 = 1.50\ \mu\text{F}$. The energy stored under these conditions is

$$(\text{Energy stored})_3 = \frac{1}{2}C_f(\Delta V)_i^2 = \frac{1}{2}(1.50\ \mu\text{F})(6.00\ \text{V})^2 = \boxed{27.0\ \mu\text{J}}$$

16.48 The energy transferred to the water is

$$W = \frac{1}{100}\left[\frac{1}{2}Q(\Delta V)\right] = \frac{(50.0\ \text{C})(1.00\times10^8\ \text{V})}{200} = 2.50\times10^7\ \text{J}$$

Thus, if m is the mass of water boiled away, $W = m\left[c(\Delta T) + L_v\right]$ becomes

$$2.50\times10^7\ \text{J} = m\left[\left(4186\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(100^\circ\text{C} - 30.0^\circ\text{C}) + 2.26\times10^6\ \text{J/kg}\right]$$

giving $$m = \frac{2.50\times10^7\ \text{J}}{\left[2.93\times10^5\ \text{J/kg} + 2.26\times10^6\ \text{J/kg}\right]} = \boxed{9.79\ \text{kg}}$$

16.49 (a) Note that the charge on the plates remains constant at the original value, Q_0, as the dielectric is inserted. Thus, the change in the potential difference, $\Delta V = Q/C$, is due to a change in capacitance alone. The ratio of the final and initial capacitances is

$$\frac{C_f}{C_i} = \frac{\kappa \epsilon_0 A/d}{\epsilon_0 A/d} = \kappa \quad \text{and} \quad \frac{C_f}{C_i} = \frac{Q_0/(\Delta V)_f}{Q_0/(\Delta V)_i} = \frac{(\Delta V)_i}{(\Delta V)_f} = \frac{85.0\ \text{V}}{25.0\ \text{V}} = 3.40$$

continued on next page

Thus, the dielectric constant of the inserted material is $\boxed{\kappa = 3.40}$, and the material is probably $\boxed{\text{nylon}}$ (see Table 16.1).

(b) If the dielectric only partially filled the space between the plates, leaving the remaining space air-filled, the equivalent dielectric constant would be somewhere between $\kappa = 1.00$ (air) and $\kappa = 3.40$. The resulting potential difference would then lie somewhere between $(\Delta V)_i = 85.0$ V and $(\Delta V)_f = 25.0$ V.

16.50 (a) If the maximum electric field that can exist between the plates before breakdown (i.e., the dielectric strength) is E_{max}, the maximum potential difference across the plates is $\Delta V_{max} = E_{max} \cdot d$, where d is the plate separation. The maximum change on either plate then has magnitude

$$Q_{max} = C(\Delta V_{max}) = C(E_{max} \cdot d)$$

Since the capacitance of a parallel-plate capacitor is $C = \kappa \in_0 A/d$, the maximum charge is

$$Q_{max} = \left(\frac{\kappa \in_0 A}{\not{d}}\right)(E_{max} \cdot \not{d}) = \kappa \in_0 AE_{max}$$

The area of each plate is $A = 5.00 \text{ cm}^2 = 5.00 \times 10^{-4} \text{ m}^2$, and when air is the dielectric, $\kappa = 1.00$ and $E_{max} = 3.00 \times 10^6$ V/m (see Table 16.1). Thus,

$$Q_{max} = (1.00)(8.85 \times 10^{-12} \text{ C/N} \cdot \text{m}^2)(5.00 \times 10^{-4} \text{ m}^2)(3.00 \times 10^6 \text{ V/m})$$

$$= 1.33 \times 10^{-8} \text{ C} = \boxed{13.3 \text{ nC}}$$

(b) If the dielectric is now polystyrene $(\kappa = 2.56 \text{ and } E_{max} = 24.0 \times 10^6 \text{ V/m})$, then

$$Q_{max} = (2.56)(8.85 \times 10^{-12} \text{ C/N} \cdot \text{m}^2)(5.00 \times 10^{-4} \text{ m}^2)(24.0 \times 10^6 \text{ V/m})$$

$$= 2.72 \times 10^{-7} \text{ C} = \boxed{272 \text{ nC}}$$

16.51 (a) The dielectric constant for Teflon® is $\kappa = 2.1$, so the capacitance is

$$C = \frac{\kappa \in_0 A}{d} = \frac{(2.1)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(175 \times 10^{-4} \text{ m}^2)}{0.040\,0 \times 10^{-3} \text{ m}}$$

$$C = 8.1 \times 10^{-9} \text{ F} = \boxed{8.1 \text{ nF}}$$

(b) For Teflon®, the dielectric strength is $E_{max} = 60 \times 10^6$ V/m, so the maximum voltage is

$$\Delta V_{max} = E_{max} d = (60 \times 10^6 \text{ V/m})(0.040\,0 \times 10^{-3} \text{ m})$$

$$\Delta V_{max} = 2.4 \times 10^3 \text{ V} = \boxed{2.4 \text{ kV}}$$

16.52 Before the capacitor is rolled, the capacitance of this parallel-plate capacitor is

$$C = \frac{\kappa \in_0 A}{d} = \frac{\kappa \in_0 (w \times L)}{d}$$

where A is the surface area of one side of a foil strip. Thus, the required length is

$$L = \frac{C \cdot d}{\kappa \in_0 w} = \frac{(9.50 \times 10^{-8} \text{ F})(0.025\,0 \times 10^{-3} \text{ m})}{(3.70)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(7.00 \times 10^{-2} \text{ m})} = \boxed{1.04 \text{ m}}$$

16.53 (a) $V = \dfrac{m}{\rho} = \dfrac{1.00\times10^{-12}\ \text{kg}}{1100\ \text{kg/m}^3} = \boxed{9.09\times10^{-16}\ \text{m}^3}$

Since $V = 4\pi r^3/3$, the radius is $r = \left[3V/4\pi\right]^{1/3}$, and the surface area is

$$A = 4\pi r^2 = 4\pi\left[\frac{3V}{4\pi}\right]^{2/3} = 4\pi\left[\frac{3\left(9.09\times10^{-16}\ \text{m}^3\right)}{4\pi}\right]^{2/3} = \boxed{4.54\times10^{-10}\ \text{m}^2}$$

(b) $C = \dfrac{\kappa \in_0 A}{d} = \dfrac{(5.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.54\times10^{-10}\ \text{m}^2\right)}{100\times10^{-9}\ \text{m}} = \boxed{2.01\times10^{-13}\ \text{F}}$

(c) $Q = C(\Delta V) = \left(2.01\times10^{-13}\ \text{F}\right)\left(100\times10^{-3}\ \text{V}\right) = \boxed{2.01\times10^{-14}\ \text{C}}$

and the number of electronic charges is

$$n = \frac{Q}{e} = \frac{2.01\times10^{-14}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = \boxed{1.26\times10^{5}}$$

16.54 For a parallel-plate capacitor, $C = \kappa \in_0 A/d$ and $Q = \sigma A = C(\Delta V)$. Thus, $\sigma A = \left(\kappa \in_0 A/d\right)(\Delta V)$, and $d = \left(\kappa \in_0/\sigma\right)(\Delta V)$. With air as the dielectric material $(\kappa = 1.00)$, the separation of the plates must be

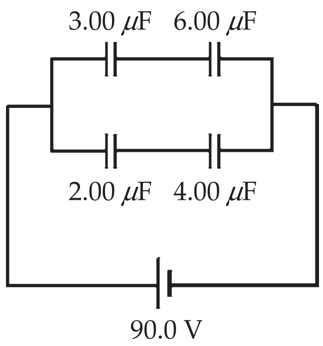
$$d = \frac{(1.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(150\ \text{V})}{\left(3.00\times10^{-10}\ \text{C/cm}^2\right)\left(10^4\ \text{cm}^2/1\ \text{m}^2\right)} = 4.43\times10^{-4}\ \text{m} = \boxed{0.443\ \text{mm}}$$

16.55 Since the capacitors are in series, the equivalent capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d_1}{\in_0 A} + \frac{d_2}{\in_0 A} + \frac{d_3}{\in_0 A} = \frac{d_1 + d_2 + d_3}{\in_0 A}$$

or $C_{eq} = \boxed{\dfrac{\in_0 A}{d}\ \text{where}\ d = d_1 + d_2 + d_3}$

16.56 (a) Please refer to the solution of Problem 16.37 where the following results were obtained:



$C_{eq} = 3.33\ \mu\text{F}$ $\quad Q_3 = Q_6 = 180\ \mu\text{C}$ $\quad Q_2 = Q_4 = 120\ \mu\text{C}$

The total energy stored in the full circuit is then

$$(\text{Energy stored})_{\text{total}} = \frac{1}{2}C_{eq}(\Delta V)^2 = \frac{1}{2}\left(3.33\times10^{-6}\ \text{F}\right)(90.0\ \text{V})^2$$

$$= 1.35\times10^{-2}\ \text{J} = 13.5\times10^{-3}\ \text{J} = \boxed{13.5\ \text{mJ}}$$

(b) The energy stored in each individual capacitor is

For $2.00\ \mu\text{F}$: $\quad (\text{Energy stored})_2 = \dfrac{Q_2^2}{2C_2} = \dfrac{\left(120\times10^{-6}\ \text{C}\right)^2}{2\left(2.00\times10^{-6}\ \text{F}\right)} = 3.60\times10^{-3}\ \text{J} = \boxed{3.60\ \text{mJ}}$

For $3.00\ \mu\text{F}$: $\quad (\text{Energy stored})_3 = \dfrac{Q_3^2}{2C_3} = \dfrac{\left(180\times10^{-6}\ \text{C}\right)^2}{2\left(3.00\times10^{-6}\ \text{F}\right)} = 5.40\times10^{-3}\ \text{J} = \boxed{5.40\ \text{mJ}}$

continued on next page

68719_16_ch16_p030-061.indd 56 1/7/11 2:33:39 PM

For 4.00 μF: $\quad (\text{Energy stored})_4 = \dfrac{Q_4^2}{2C_4} = \dfrac{(120\times10^{-6}\text{ C})^2}{2(4.00\times10^{-6}\text{ F})} = 1.80\times10^{-3}\text{ J} = \boxed{1.80\text{ mJ}}$

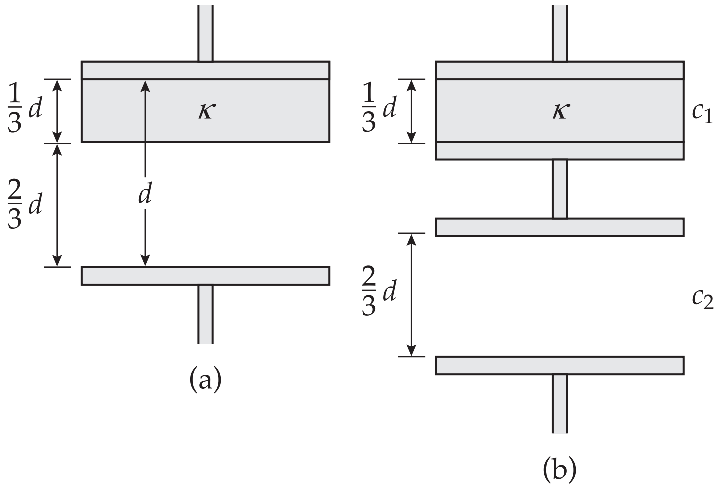
For 6.00 μF: $\quad (\text{Energy stored})_6 = \dfrac{Q_6^2}{2C_6} = \dfrac{(180\times10^{-6}\text{ C})^2}{2(6.00\times10^{-6}\text{ F})} = 2.70\times10^{-3}\text{ J} = \boxed{2.70\text{ mJ}}$

(c) The total energy stored in the individual capacitors is

$$\text{Energy stored} = (3.60+5.40+1.80+2.70)\text{ mJ} = \boxed{13.5\text{ mJ}} = (\text{Energy stored})_{\text{total}}$$

Thus, the sums of the energies stored in the individual capacitors equals the total energy stored by the system.

16.57 In the absence of a dielectric, the capacitance of the parallel-plate capacitor is $C_0 = \epsilon_0 A/d$.



With the dielectric inserted, it fills one-third of the gap between the plates as shown in sketch (a) at the right. We model this situation as consisting of a pair of capacitors, C_1 and C_2, connected in series as shown in sketch (b) at the right. In reality, the lower plate of C_1 and the upper plate of C_2 are one and the same, consisting of the lower surface of the dielectric shown in sketch (a). The capacitances in the model of sketch (b) are given by

$$C_1 = \frac{\kappa \epsilon_0 A}{d/3} = \frac{3\kappa \epsilon_0 A}{d} \quad \text{and} \quad C_2 = \frac{\epsilon_0 A}{2d/3} = \frac{3\epsilon_0 A}{2d}$$

The equivalent capacitance of the series combination is

$$\frac{1}{C_{\text{eq}}} = \frac{d}{3\kappa\epsilon_0 A} + \frac{2d}{3\epsilon_0 A} = \left(\frac{1}{\kappa}+2\right)\left(\frac{d}{3\epsilon_0 A}\right) = \left(\frac{2\kappa+1}{\kappa}\right)\frac{d}{3\epsilon_0 A} = \left(\frac{2\kappa+1}{3\kappa}\right)\frac{d}{\epsilon_0 A} = \left(\frac{2\kappa+1}{3\kappa}\right)\frac{1}{C_0}$$

and $\boxed{C_{\text{eq}} = [3\kappa/(2\kappa+1)]C_0}$.

16.58 For the parallel combination: $C_p = C_1 + C_2$ which gives $\quad C_2 = C_p - C_1$ **[1]**

For the series combination: $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \quad$ or $\quad \dfrac{1}{C_2} = \dfrac{1}{C_s} - \dfrac{1}{C_1} = \dfrac{C_1 - C_s}{C_s C_1}$

Thus, we have $C_2 = \dfrac{C_s C_1}{C_1 - C_s}$ and equating this to Equation [1] above gives

$$C_p - C_1 = \frac{C_s C_1}{C_1 - C_s} \quad \text{or} \quad C_p C_1 - C_p C_s - C_1^2 + \cancel{C_s C_1} = \cancel{C_s C_1}$$

We write this result as: $\quad C_1^2 - C_p C_1 + C_p C_s = 0$

continued on next page

68719_16_ch16_p030-061.indd 57 1/7/11 2:33:42 PM

and use the quadratic formula to obtain $\boxed{C_1 = \frac{1}{2}C_p \pm \sqrt{\frac{1}{4}C_p^2 - C_p C_s}}$

Then, Equation [1] gives $\boxed{C_2 = \frac{1}{2}C_p \mp \sqrt{\frac{1}{4}C_p^2 - C_p C_s}}$

16.59 For a parallel-plate capacitor with plate separation d,

$$\Delta V_{\max} = E_{\max} \cdot d \qquad \text{or} \qquad d = \frac{\Delta V_{\max}}{E_{\max}}$$

The capacitance is then

$$C = \frac{\kappa \in_0 A}{d} = \kappa \in_0 A\left(\frac{E_{\max}}{\Delta V_{\max}}\right)$$

and the needed area of the plates is $A = C \cdot \Delta V_{\max} / \kappa \in_0 E_{\max}$, or

$$A = \frac{(0.250\times10^{-6}\text{ F})(4.00\times10^{3}\text{ V})}{(3.00)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.00\times10^{8}\text{ V/m})} = \boxed{0.188\text{ m}^2}$$

16.60 (a) The 1.0-μC is located 0.50 m from point P, so its contribution to the potential at P is

$$V_1 = k_e\frac{q_1}{r_1} = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{1.0\times10^{-6}\text{ C}}{0.50\text{ m}}\right) = \boxed{1.8\times10^{4}\text{ V}}$$

(b) The potential at P due to the –2.0-μC charge located 0.50 m away is

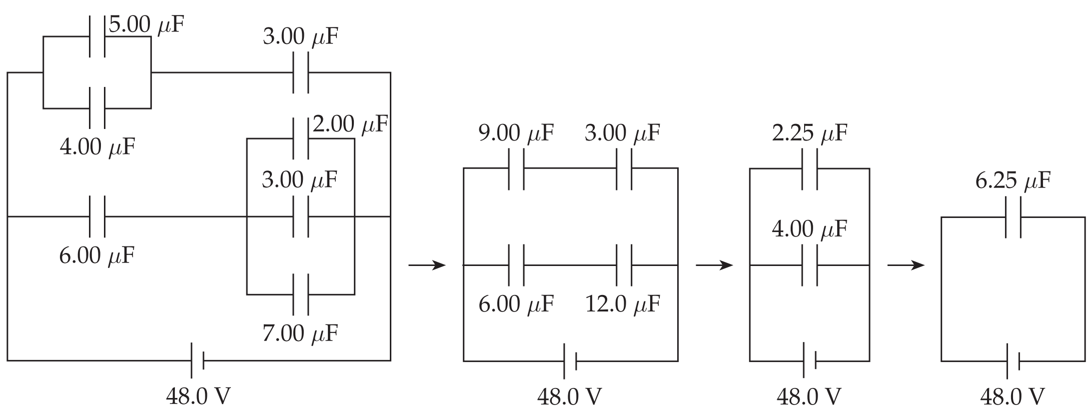
$$V_2 = k_e\frac{q_2}{r_2} = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-2.0\times10^{-6}\text{ C}}{0.50\text{ m}}\right) = \boxed{-3.6\times10^{4}\text{ V}}$$

(c) The total potential at point P is $V_P = V_1 + V_2 = (+1.8 - 3.6)\times10^{4}\text{ V} = \boxed{-1.8\times10^{4}\text{ V}}$

(d) The work required to move a charge $q = 3.0\ \mu$C to point P from infinity is

$$W = q\Delta V = q(V_P - V_\infty) = (3.0\times10^{-6}\text{ C})(-1.8\times10^{4}\text{ V} - 0) = \boxed{-5.4\times10^{-2}\text{ J}}$$

16.61 The stages for the reduction of this circuit are shown below.



Thus, $C_{eq} = \boxed{6.25\ \mu\text{F}}$

16.62 (a) Due to spherical symmetry, the charge on each of the concentric spherical shells will be uniformly distributed over that shell. Inside a spherical surface having a uniform charge distribution, the electric field due to the charge on that surface is zero. Thus, in this region, the potential due to the charge on that surface is constant and equal to the potential at the surface. Outside a spherical surface having a uniform charge distribution, the potential due to the charge on that surface is given by $V = k_e q/r$, where r is the distance from the center of that surface and q is the charge on that surface.

In the region between a pair of concentric spherical shells, with the inner shell having charge $+Q$ and the outer shell having radius b and charge $-Q$, the total electric potential at distance r from the center is given by

$$V = V_{\substack{\text{due to}\\ \text{inner shell}}} + V_{\substack{\text{due to}\\ \text{outer shell}}} = \frac{k_e Q}{r} + \frac{k_e(-Q)}{b} = k_e Q\left(\frac{1}{r} - \frac{1}{b}\right)$$

The potential difference between the two shells is therefore

$$\Delta V = V\big|_{r=a} - V\big|_{r=b} = k_e Q\left(\frac{1}{a} - \frac{1}{b}\right) - k_e Q\left(\frac{1}{b} - \frac{1}{b}\right) = k_e Q\left(\frac{b-a}{ab}\right)$$

The capacitance of this device is given by

$$C = \frac{Q}{\Delta V} = \boxed{\frac{ab}{k_e(b-a)}}$$

(b) When $b >> a$, then $b - a \approx b$. Thus, in the limit as $b \to \infty$, the capacitance found above becomes

$$C \to \frac{ab}{k_e(b)} = \frac{a}{k_e} = \boxed{4\pi \epsilon_0 \, a}$$

16.63 The energy stored in a charged capacitor is $E_{\text{stored}} = \frac{1}{2}C(\Delta V)^2$. Hence,

$$\Delta V = \sqrt{\frac{2E_{\text{stored}}}{C}} = \sqrt{\frac{2(300\text{ J})}{30.0\times 10^{-6}\text{ F}}} = 4.47\times 10^3\text{ V} = \boxed{4.47\text{ kV}}$$

16.64 From $Q = C(\Delta V)$, the capacitance of the capacitor with air between the plates is

$$C_0 = \frac{Q_0}{\Delta V} = \frac{150\ \mu\text{C}}{\Delta V}$$

After the dielectric is inserted, the potential difference is held to the original value, but the charge changes to $Q = Q_0 + 200\ \mu\text{C} = 350\ \mu\text{C}$. Thus, the capacitance with the dielectric slab in place is

$$C = \frac{Q}{\Delta V} = \frac{350\ \mu\text{C}}{\Delta V}$$

The dielectric constant of the dielectric slab is therefore

$$\kappa = \frac{C}{C_0} = \left(\frac{350\ \mu\text{C}}{\cancel{\Delta V}}\right)\left(\frac{\cancel{\Delta V}}{150\ \mu\text{C}}\right) = \frac{350}{150} = \boxed{2.33}$$

16.65 The charges initially stored on the capacitors are

$$Q_1 = C_1(\Delta V)_i = (6.0\ \mu\text{F})(250\ \text{V}) = 1.5\times 10^3\ \mu\text{C}$$

and $$Q_2 = C_2(\Delta V)_i = (2.0\ \mu\text{F})(250\ \text{V}) = 5.0\times 10^2\ \mu\text{C}$$

When the capacitors are connected in parallel, with the negative plate of one connected to the positive pl ate of the other, the net stored charge is

$$Q = Q_1 - Q_2 = 1.5\times 10^3\ \mu\text{C} - 5.0\times 10^2\ \mu\text{C} = 1.0\times 10^3\ \mu\text{C}$$

The equivalent capacitance of the parallel combination is $C_{\text{eq}} = C_1 + C_2 = 8.0\ \mu\text{F}$. Thus, the final potential difference across each of the capacitors is

$$(\Delta V)' = \frac{Q}{C_{\text{eq}}} = \frac{1.0\times 10^3\ \mu\text{C}}{8.0\ \mu\text{F}} = 125\ \text{V}$$

and the final charge on each capacitor is

$$Q_1' = C_1(\Delta V)' = (6.0\ \mu\text{F})(125\ \text{V}) = 750\ \mu\text{C} = \boxed{0.75\ \text{mC}}$$

and $$Q_2' = C_2(\Delta V)' = (2.0\ \mu\text{F})(125\ \text{V}) = 250\ \mu\text{C} = \boxed{0.25\ \text{mC}}$$

16.66 (a) The distance from the charge $2q$ to either of the charges on the y-axis is $r = \sqrt{d^2 + (2d)^2} = \sqrt{5}\,d$. Thus,

$$V = \sum_i \frac{k_e q_i}{r_i} = \frac{k_e q}{\sqrt{5}\,d} + \frac{k_e q}{\sqrt{5}\,d} = \boxed{\frac{2k_e q}{\sqrt{5}\,d}}$$

(b) $$PE_{2q} = \frac{k_e q_1 q_3}{r_1} + \frac{k_e q_2 q_3}{r_2} = \frac{k_e q(2q)}{\sqrt{5}\,d} + \frac{k_e q(2q)}{\sqrt{5}\,d} = \boxed{\frac{4k_e q^2}{\sqrt{5}\,d}}$$

(c) From conservation of energy with $PE = 0$ at $r = \infty$,

$$KE_f = KE_i + PE_i - PE_f = 0 + \frac{4k_e q^2}{\sqrt{5}\,d} - 0 = \boxed{\frac{4k_e q^2}{\sqrt{5}\,d}}$$

(d) $$v_f = \sqrt{\frac{2(KE_f)}{m}} = \sqrt{\frac{2}{m}\left(\frac{4k_e q^2}{\sqrt{5}\,d}\right)} = \boxed{\left(\frac{8k_e q^2}{\sqrt{5}\,md}\right)^{\frac{1}{2}}}$$

16.67 When excess charge resides on a spherical surface that is far removed from any other charge, this excess charge is uniformly distributed over the spherical surface, and the electric potential at the surface is the same as if all the excess charge were concentrated at the center of the spherical surface.

In the given situation, we have two charged spheres, initially isolated from each other, with charges and potentials of $Q_A = +6.00\ \mu\text{C}$ and $V_A = k_e Q_A/R_A$, where $R_A = 12.0$ cm, $Q_B = -4.00\ \mu\text{C}$, and $V_B = k_e Q_B/R_B$, with $R_B = 18.0$ cm.

When these spheres are then connected by a long conducting thread, the charges are redistributed (yielding charges of Q_A' and Q_B', respectively) until the two surfaces come to a common potential $\left(V_A' = kQ_A'/R_A = V_B' = kQ_B'/R_B\right)$. When equilibrium is established, we have:

From conservation of charge: $Q_A' + Q_B' = Q_A + Q_B \quad \Rightarrow \quad Q_A' + Q_B' = +2.00\ \mu\text{C}$ **[1]**

continued on next page

From equal potentials: $\frac{kQ'_A}{R_A} = \frac{kQ'_B}{R_B} \quad \Rightarrow \quad Q'_B = \left(\frac{R_B}{R_B}\right)Q'_A$ or $Q'_B = 1.50\,Q'_A$ **[2]**

Substituting Equation [2] into [1] gives $Q'_A = \frac{+2.00\ \mu\text{C}}{2.50} = \boxed{0.800\ \mu\text{C}}$

Then, Equation [2] gives $Q'_B = 1.50(0.800\ \mu\text{C}) = \boxed{1.20\ \mu\text{C}}$

16.68 The electric field between the plates is directed downward with magnitude

$$|E_y| = \frac{\Delta V}{D} = \frac{100\ \text{V}}{2.0\times10^{-3}\ \text{m}} = 5.0\times10^{4}\ \text{N/m}$$

Since the gravitational force experienced by the electron is negligible in comparison to the electrical force acting on it, the vertical acceleration is

$$a_y = \frac{F_y}{m_e} = \frac{qE_y}{m_e} = \frac{(-1.60\times10^{-19}\ \text{C})(-5.0\times10^{4}\ \text{N/m})}{9.11\times10^{-31}\ \text{kg}} = +8.8\times10^{15}\ \text{m/s}^2$$

(a) At the closest approach to the bottom plate, $v_y = 0$. Thus, the vertical displacement from point O is found from $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ as

$$\Delta y = \frac{0-(v_0 \sin\theta_0)^2}{2a_y} = \frac{-\left[-(5.6\times10^{6}\ \text{m/s})\sin 45°\right]^2}{2(8.8\times10^{15}\ \text{m/s}^2)} = -8.9\times10^{-4}\ \text{m} = -0.89\ \text{mm}$$

The minimum distance above the bottom plate is then

$$d = \frac{D}{2} + \Delta y = 1.0\ \text{mm} - 0.89\ \text{mm} = \boxed{0.1\ \text{mm}}$$

(b) The time for the electron to go from point O to the upper plate ($\Delta y = +1.0$ mm) is found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ as

$$+1.0\times10^{-3}\ \text{m} = \left[-\left(5.6\times10^{6}\ \frac{\text{m}}{\text{s}}\right)\sin 45°\right]t + \frac{1}{2}\left(8.8\times10^{15}\ \frac{\text{m}}{\text{s}^2}\right)t^2$$

Solving for t gives a positive solution of $t = 1.1\times10^{-9}$ s. The horizontal displacement from point O at this time is

$$\Delta x = v_{0x}t = \left[(5.6\times10^{6}\ \text{m/s})\cos 45°\right](1.1\times10^{-9}\ \text{s}) = \boxed{4.4\ \text{mm}}$$