13-1 What is Physics?

One of the long-standing goals of physics is to understand the gravitational force—the force that holds you to Earth, holds the Moon in orbit around Earth, and holds Earth in orbit around the Sun. It also reaches out through the whole of our Milky Way galaxy, holding together the billions and billions of stars in the Galaxy and the countless molecules and dust particles between stars. We are located somewhat near the edge of this disk-shaped collection of stars and other matter, $2.6 \times 10^4$ light-years ($2.5 \times 10^{20}$ m) from the galactic center, around which we slowly revolve.

The gravitational force also reaches across intergalactic space, holding together the Local Group of galaxies, which includes, in addition to the Milky Way, the Andromeda Galaxy (Fig. 13-1) at a distance of $2.3 \times 10^6$ light-years away from Earth, plus several closer dwarf galaxies, such as the Large Magellanic Cloud. The Local Group is part of the Local Supercluster of galaxies that is being drawn by the gravitational force toward an exceptionally massive region of space called the Great Attractor. This region appears to be about $3.0 \times 10^8$ light-years from Earth, on the opposite side of the Milky Way. And the gravitational force is even more far-reaching because it attempts to hold together the entire universe, which is expanding.
The Andromeda Galaxy. Located $2.3 \times 10^6$ light-years from us, and faintly visible to the naked eye, it is very similar to our home galaxy, the Milky Way. (Courtesy NASA)

This force is also responsible for some of the most mysterious structures in the universe: black holes. When a star considerably larger than our Sun burns out, the gravitational force between all its particles can cause the star to collapse in on itself and thereby to form a black hole. The gravitational force at the surface of such a collapsed star is so strong that neither particles nor light can escape from the surface (thus the term “black hole”). Any star coming too near a black hole can be ripped apart by the strong gravitational force and pulled into the hole. Enough captures like this yields a supermassive black hole. Such mysterious monsters appear to be common in the universe.

Although the gravitational force is still not fully understood, the starting point in our understanding of it lies in the law of gravitation of Isaac Newton.

**Newton's Law of Gravitation**

Physicists like to study seemingly unrelated phenomena to show that a relationship can be found if the phenomena are examined closely enough. This search for unification has been going on for centuries. In 1665, the 23-year-old Isaac Newton made a basic contribution to physics when he showed that the force that holds the Moon in its orbit is the same force that makes an apple fall. We take this knowledge so much for granted now that it is not easy for us to comprehend the ancient belief that the motions of earthbound bodies and heavenly bodies were different in kind and were governed by different laws.
Newton concluded not only that Earth attracts both apples and the Moon but also that every body in the universe attracts every other body; this tendency of bodies to move toward one another is called gravitation. Newton’s conclusion takes a little getting used to, because the familiar attraction of Earth for earth-bound bodies is so great that it overwhelms the attraction that earthbound bodies have for each other. For example, Earth attracts an apple with a force magnitude of about 0.8 N. You also attract a nearby apple (and it attracts you), but the force of attraction has less magnitude than the weight of a speck of dust.

Newton proposed a force law that we call Newton’s law of gravitation: Every particle attracts any other particle with a gravitational force of magnitude

$$ F = G \frac{m_1 m_2}{r^2} $$

(Newton’s law of gravitation) (13-1)

Here $m_1$ and $m_2$ are the masses of the particles, $r$ is the distance between them, and $G$ is the gravitational constant, with a value that is now known to be

$$ G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 $$

(13-2)

In Fig. 13-2a, $\vec{F}$ is the gravitational force acting on particle 1 (mass $m_1$) due to particle 2 (mass $m_2$). The force is directed toward particle 2 and is said to be an attractive force because particle 1 is attracted toward particle 2. The magnitude of the force is given by Eq. 13-1.

We can describe $\vec{F}$ as being in the positive direction of an $r$ axis extending radially from particle 1 through particle 2 (Fig. 13-2b). We can also describe $\vec{F}$ by using a radial unit vector $\hat{r}$ (a dimensionless vector of magnitude 1) that is directed away from particle 1 along the $r$ axis (Fig. 13-2c). From Eq. 13-1, the force on particle 1 is then

$$ \vec{F} = \hat{r} \cdot G \frac{m_1 m_2}{r^2} $$

(13-3)

The gravitational force on particle 2 due to particle 1 has the same magnitude as the force on particle 1 but the opposite direction. These two forces form a third-law force pair, and we can speak of the gravitational force between the two particles as having a magnitude given by Eq. 13-1. This force between two particles is not altered by other objects, even if they are located between the particles. Put another way, no object can shield either particle from the gravitational force due to the other particle.

The strength of the gravitational force—that is, how strongly two particles with given masses at a given separation attract each other—depends on the value of the gravitational constant $G$. If $G$—by some miracle—were suddenly multiplied by a factor of
10, you would be crushed to the floor by Earth's attraction. If $G$ were divided by this factor, Earth's attraction would be so weak that you could jump over a building.

Although Newton's law of gravitation applies strictly to particles, we can also apply it to real objects as long as the sizes of the objects are small relative to the distance between them. The Moon and Earth are far enough apart so that, to a good approximation, we can treat them both as particles—but what about an apple and Earth? From the point of view of the apple, the broad and level Earth, stretching out to the horizon beneath the apple, certainly does not look like a particle.

Newton solved the apple–Earth problem by proving an important theorem called the shell theorem:

Earth can be thought of as a nest of such shells, one within another and each shell attracting a particle outside Earth's surface as if the mass of that shell were at the center of the shell. Thus, from the apple's point of view, Earth does behave like a particle, one that is located at the center of Earth and has a mass equal to that of Earth.

Suppose that, as in Fig. 13-3, Earth pulls down on an apple with a force of magnitude 0.80 N. The apple must then pull up on Earth with a force of magnitude 0.80 N, which we take to act at the center of Earth. Although the forces are matched in magnitude, they produce different accelerations when the apple is released. The acceleration of the apple is about 9.8 m/s$^2$, the familiar acceleration of a falling body near Earth's surface. The acceleration of Earth, however, measured in a reference frame attached to the center of mass of the apple–Earth system, is only about $1 \times 10^{-25}$ m/s$^2$.

![Figure 13-3](https://example.com/fig13-3.png)

**Figure 13-3** The apple pulls up on Earth just as hard as Earth pulls down on the apple.

**CHECKPOINT 1**

A particle is to be placed, in turn, outside four objects, each of mass $m$: (1) a large uniform solid sphere, (2) a large uniform spherical shell, (3) a small uniform solid sphere, and (4) a small uniform shell. In each situation, the distance between the particle and the center of the object is $d$. Rank the objects according to the magnitude of the gravitational force they exert on the particle, greatest first.
Gravitation and the Principle of Superposition

Given a group of particles, we find the net (or resultant) gravitational force on any one of them from the others by using the **principle of superposition**. This is a general principle that says a net effect is the sum of the individual effects. Here, the principle means that we first compute the individual gravitational forces that act on our selected particle due to each of the other particles. We then find the net force by adding these forces vectorially, as usual.

For \( n \) interacting particles, we can write the principle of superposition for the gravitational forces on particle 1 as

\[
\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \ldots + \vec{F}_{1n}.
\]  

(13-4)

Here \( \vec{F}_{1,\text{net}} \) is the net force on particle 1 due to the other particles and, for example, \( \vec{F}_{13} \) is the force on particle 1 from particle 3. We can express this equation more compactly as a vector sum:

\[
\vec{F}_{1,\text{net}} = \sum_{i=2}^{n} \vec{F}_{1i}.
\]  

(13-5)

What about the gravitational force on a particle from a real (extended) object? This force is found by dividing the object into parts small enough to treat as particles and then using Eq. (13-5) to find the vector sum of the forces on the particle from all the parts. In the limiting case, we can divide the extended object into differential parts each of mass \( dm \) and each producing a differential force \( d\vec{F} \) on the particle. In this limit, the sum of Eq. (13-5) becomes an integral and we have

\[
\vec{F}_1 = \int d\vec{F},
\]  

(13-6)

in which the integral is taken over the entire extended object and we drop the subscript “net.” If the extended object is a uniform sphere or a spherical shell, we can avoid the integration of Eq. (13-6) by assuming that the object's mass is concentrated at the object's center and using Eq. (13-1).

---

**CHECKPOINT 2**

The figure shows four arrangements of three particles of equal masses. (a) Rank the arrangements according to the magnitude of the net gravitational force on the particle labeled \( m \), greatest first. (b) In arrangement 2, is the direction of the net force closer to the line of length \( d \) or to the line of length \( D \)?
Net gravitational force, 2D, 3 particles

Figure 13-4a shows an arrangement of three particles, particle 1 of mass \( m_1 = 6.0 \) kg and particles 2 and 3 of mass \( m_2 = m_3 = 4.0 \) kg, and distance \( a = 2.0 \) cm. What is the net gravitational force \( \vec{F}_{\text{net}} \) on particle 1 due to the other particles?

We want the forces (pulls) on particle 1, not the forces on the other particles.

This is the force (pull) on particle 1 due to particle 2.

This is the force (pull) on particle 1 due to particle 3.

This is one way to show the net force on particle 1. Note the head-to-tail arrangement.

This is another way, also a head-to-tail arrangement.

A calculator’s inverse tangent can give this for the angle.

But this is correct an

Figure 13-4a) An arrangement of three particles. The force on particle 1 due to (b) particle 2 and (c) particle 3. (d) – (g) Ways to combine the forces to get the net force magnitude and orientation.
(1) Because we have particles, the magnitude of the gravitational force on particle 1 due to either of the other particles is given by Eq. 13-1 \((F = Gm_1m_2/r^2)\). (2) The direction of either gravitational force on particle 1 is toward the particle responsible for it. (3) Because the forces are not along a single axis, we cannot simply add or subtract their magnitudes or their components to get the net force. Instead, we must add them as vectors.

**Calculations:**

From Eq. 13-1, the magnitude of the force \(\vec{F}_{12}\) on particle 1 from particle 2 is

\[
\vec{F}_{12} = \frac{Gm_1m_2}{a^2} = \frac{6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2}{0.023 \text{ m}^2} (6.0 \text{ kg}) (4.0 \text{ kg}) \]

\[
= 4.00 \times 10^{-5} \text{ N} \tag{13-7}
\]

Similarly, the magnitude of force \(\vec{F}_{13}\) on particle 1 from particle 3 is

\[
\vec{F}_{13} = \frac{Gm_1m_3}{2a^2} = \frac{6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2}{0.04 \text{ m}^2} (6.0 \text{ kg}) (4.0 \text{ kg}) \]

\[
= 1.00 \times 10^{-5} \text{ N} \tag{13-8}
\]

Force \(\vec{F}_{12}\) is directed in the positive direction of the \(y\) axis (Fig. 13-4b) and has only the \(y\) component \(F_{12}\). Similarly, \(\vec{F}_{13}\) is directed in the negative direction of the \(x\) axis and has only the \(x\) component \(-F_{13}\) (Fig. 13-4c).

To find the net force \(\vec{F}_{1, \text{net}}\) on particle 1, we must add the two forces as vectors (Figs. 13-4d and e). We can do so on a vector-capable calculator. However, here we note that \(-F_{13}\) and \(F_{12}\) are actually the \(x\) and \(y\) components of \(\vec{F}_{1, \text{net}}\). Therefore, we can use Eq. 3-6 to find first the magnitude and then the direction of \(\vec{F}_{1, \text{net}}\). The magnitude is

\[
F_{1, \text{net}} = \sqrt{(F_{12})^2 + (-F_{13})^2} = \sqrt{(4.00 \times 10^{-5} \text{ N})^2 + (-1.00 \times 10^{-5} \text{ N})^2} \]

\[
= 4.1 \times 10^{-6} \text{ N}. \tag{Answer}
\]

Relative to the positive direction of the \(x\) axis, Eq. 3-6 gives the direction of \(\vec{F}_{1, \text{net}}\) as

\[
\theta = \tan^{-1} \left( \frac{F_{12}}{-F_{13}} \right) = \tan^{-1} \left( \frac{4.00 \times 10^{-6} \text{ N}}{-1.00 \times 10^{-5} \text{ N}} \right) = -76^\circ.
\]

Is this a reasonable direction (Fig. 13-4f)? No, because the direction of \(\vec{F}_{1, \text{net}}\) must be between the directions of
Recall from Chapter 3 that a calculator displays only one of the two possible answers to a \( \tan^{-1} \) function. We find the other answer by adding 180°:

\[
-76^\circ + 180^\circ = 104^\circ, \quad \text{(Answer)}
\]

which is a reasonable direction for \( \mathbf{F}_{1, \text{net}} \) (Fig. 13-4g).

---

13-4 Gravitation Near Earth’s Surface

Let us assume that Earth is a uniform sphere of mass \( M \). The magnitude of the gravitational force from Earth on a particle of mass \( m \), located outside Earth a distance \( r \) from Earth’s center, is then given by Eq. 13-1 as

\[
\mathbf{F} = \frac{GMm}{r^2}.
\]

(13-9)

If the particle is released, it will fall toward the center of Earth, as a result of the gravitational force \( \mathbf{F} \), with an acceleration we shall call the gravitational acceleration \( \mathbf{a} \). Newton’s second law tells us that magnitudes \( F \) and \( a \) are related by

\[
\mathbf{F} = ma \quad \text{g}.
\]

(13-10)

Now, substituting \( F \) from Eq. 13-9 into Eq. 13-10 and solving for \( a \), we find

\[
a = \frac{GM}{r^2}.
\]

(13-11)

Table 13-1 shows values of \( a \) computed for various altitudes above Earth’s surface. Notice \( a \) is significant even at 400 km.

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>( a ) (m/s(^2))</th>
<th>Altitude Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.83</td>
<td>Mean Earth surface</td>
</tr>
<tr>
<td>8.8</td>
<td>9.80</td>
<td>Mt. Everest</td>
</tr>
<tr>
<td>36.6</td>
<td>9.71</td>
<td>Highest crewed balloon</td>
</tr>
<tr>
<td>400</td>
<td>8.70</td>
<td>Space shuttle orbit</td>
</tr>
<tr>
<td>35 700</td>
<td>0.225</td>
<td>Communications satellite.</td>
</tr>
</tbody>
</table>

Since Section 5-4, we have assumed that Earth is an inertial frame by neglecting its rotation. This simplification has allowed us to assume that the free-fall acceleration \( g \) of a particle is the same as the particle’s gravitational acceleration (which we now call \( a \)). Furthermore, we assumed that \( g \) has the constant value 9.8 m/s\(^2\) any place on Earth’s surface. However, any \( g \) value measured at a given location will differ from the \( a \) value calculated with Eq. 13-11 for that location for three reasons: (1) Earth’s mass is not distributed uniformly, (2) Earth is not a perfect sphere, and (3) Earth rotates. Moreover, because \( g \) differs from \( a \), the same three reasons mean that the measured weight \( mg \) of a particle differs from the magnitude of the gravitational force on the particle as given by Eq. 13-9. Let us now examine those reasons.

1. **Earth’s mass is not uniformly distributed.** The density (mass per unit volume) of Earth varies radially as shown in Fig. 13-5, and the density of the crust (outer section) varies from region to region over Earth’s surface. Thus, \( g \) varies from region to region over the surface.
2. **Earth is not a sphere.** Earth is approximately an ellipsoid, flattened at the poles and bulging at the equator. Its equatorial radius (from its center point out to the equator) is greater than its polar radius (from its center point out to either north or south pole) by 21 km. Thus, a point at the poles is closer to the dense core of Earth than is a point on the equator. This is one reason the free-fall acceleration \( g \) increases if you were to measure it while moving at sea level from the equator toward the north or south pole. As you move, you are actually getting closer to the center of Earth and thus, by Newton's law of gravitation, \( g \) increases.

3. **Earth is rotating.** The rotation axis runs through the north and south poles of Earth. An object located on Earth's surface anywhere except at those poles must rotate in a circle about the rotation axis and thus must have a centripetal acceleration directed toward the center of the circle. This centripetal acceleration requires a centripetal net force that is also directed toward that center.

To see how Earth's rotation causes \( g \) to differ from \( a_c \), let us analyze a simple situation in which a crate of mass \( m \) is on a scale at the equator. Figure 13-6a shows this situation as viewed from a point in space above the north pole.
Two forces act on this crate.

(a) A crate sitting on a scale at Earth's equator, as seen by an observer positioned on Earth's rotation axis at some point above the north pole. (b) A free-body diagram for the crate, with a radial $r$ axis extending from Earth's center. The gravitational force on the crate is represented with its equivalent $mg$. The normal force on the crate from the scale is directed outward, in the positive direction of the $r$ axis. The gravitational force is directed downward. Because of Earth's rotation, the crate has a centripetal acceleration $\alpha$ that is directed toward Earth's center.

Figure 13-6(a) A crate sitting on a scale at Earth's equator, as seen by an observer positioned on Earth's rotation axis at some point above the north pole. (b) A free-body diagram for the crate, with a radial $r$ axis extending from Earth's center. The gravitational force on the crate is represented with its equivalent $mg$. The normal force on the crate from the scale is $F_N$. Because of Earth's rotation, the crate has a centripetal acceleration $\alpha$ that is directed toward Earth's center.

Figure 13-6b, a free-body diagram for the crate, shows the two forces on the crate, both acting along a radial $r$ axis that extends from Earth's center. The normal force $F_N$ on the crate from the scale is directed outward, in the positive direction of the $r$ axis. The gravitational force, represented with its equivalent $mg$, is directed inward. Because it travels in a circle about the center of Earth as Earth turns, the crate has a centripetal acceleration $\alpha$ directed toward Earth's center. From Eq. 10-23 ($a_e = \omega^2 R$), we know this acceleration is equal to $\omega^2 R$, where $\omega$ is Earth's angular speed and $R$ is the circle's radius (approximately Earth's radius). Thus, we can write Newton's second law for forces along the $r$ axis ($F_{net, r} = ma_r$) as

$$F_N - mg = m\left(-\omega^2 R\right).$$

The magnitude $F_N$ of the normal force is equal to the weight $mg$ read on the scale. With $mg$ substituted for $F_N$, Eq. 13-12 gives us

$$mg = ma_g - m\left(\omega^2 R\right),$$

which is the desired result.
which says

\[(\text{measured weight}) = (\text{magnitude of gravitational force}) - (\text{mass times centripetal acceleration}).\]

Thus, the measured weight is less than the magnitude of the gravitational force on the crate, because of Earth's rotation.

To find a corresponding expression for \(g\) and \(a_g\), we cancel \(m\) from Eq. 13-13 to write

\[g = a_g - \omega^2 R,\]

which says

\[(\text{free-fall acceleration}) = (\text{gravitational acceleration}) - (\text{centripetal acceleration}).\]

Thus, the measured free-fall acceleration is less than the gravitational acceleration because of Earth's rotation.

The difference between accelerations \(g\) and \(a_e\) is equal to \(\omega^2 R\) and is greatest on the equator (for one reason, the radius of the circle traveled by the crate is greatest there). To find the difference, we can use Eq. 10-5 (\(\omega = \Delta \theta / \Delta t\)) and Earth’s radius \(R = 6.37 \times 10^6\) m. For one rotation of Earth, \(\theta\) is \(2\pi\) rad and the time period \(\Delta t\) is about 24 h. Using these values (and converting hours to seconds), we find that \(g\) is less than \(a_e\) by only about 0.034 m/s\(^2\) (small compared to 9.8 m/s\(^2\)). Therefore, neglecting the difference in accelerations \(g\) and \(a_e\) is often justified. Similarly, neglecting the difference between weight and the magnitude of the gravitational force is also often justified.

### Difference in acceleration at head and feet

(ew) An astronaut whose height \(h\) is 1.70 m floats “feet down” in an orbiting space shuttle at distance \(r = 6.77 \times 10^6\) m away from the center of Earth. What is the difference between the gravitational acceleration at her feet and at her head?

#### KEY IDEAS

We can approximate Earth as a uniform sphere of mass \(M_E\). Then, from Eq. 13-11, the gravitational acceleration at any distance \(r\) from the center of Earth is

\[a_g = \frac{GM_E}{r^2}.\]  

(13-15)

We might simply apply this equation twice, first with \(r = 6.77 \times 10^6\) m for the location of the feet and then with \(r = 6.77 \times 10^6 + 1.70\) m for the location of the head. However, a calculator may give us the same value for \(a_e\) twice, and thus a difference of zero, because \(h\) is so much smaller than \(r\). Here’s a more promising approach: Because we have a differential change \(dr\) in \(r\) between the astronaut’s feet and head, we should differentiate Eq. 13-15 with respect to \(r\).

#### Calculations:

The differentiation gives us

\[d a_g = -2 \frac{GM_E}{r^3} d r,\]

(13-16)

where \(d a_g\) is the differential change in the gravitational acceleration due to the differential change \(dr\) in \(r\). For the astronaut, \(dr = h\) and \(r = 6.77 \times 10^6\) m. Substituting data into Eq. 13-16, we find
where the $M_E$ value is taken from Appendix C. This result means that the gravitational acceleration of the astronaut's feet toward Earth is slightly greater than the gravitational acceleration of her head toward Earth. This difference in acceleration (often called a tidal effect) tends to stretch her body, but the difference is so small that she would never even sense the stretching, much less suffer pain from it.

**Calculations:**

We again have a differential change $dr$ in $r$ between the astronaut's feet and head, so we can again use Eq. 13-16. However, now we substitute $M_h = 1.99 \times 10^{31}$ kg for $M_E$. We find

$$\Delta g = -2\left(\frac{6.67 \times 10^{-11} \text{ m}^3 / \text{kg} \cdot \text{s}^2}{6.77 \times 10^6 \text{ m}}\right)^3(1.70 \text{ m})$$

$$= -4.37 \times 10^{-6} \text{ m/s}^2,$$

This means that the gravitational acceleration of the astronaut's feet toward the black hole is noticeably larger than that of her head. The resulting tendency to stretch her body would be bearable but quite painful. If she drifted closer to the black hole, the stretching tendency would increase drastically.

**Gravitation Inside Earth**

Newton's shell theorem can also be applied to a situation in which a particle is located inside a uniform shell, to show the following:

A uniform shell of matter exerts no net gravitational force on a particle located inside it.

**Caution:** This statement does not mean that the gravitational forces on the particle from the various elements of the shell magically disappear. Rather, it means that the sum of the force vectors on the particle from all the elements is zero.

If Earth's mass were uniformly distributed, the gravitational force acting on a particle would be a maximum at Earth's surface and would decrease as the particle moved outward, away from the planet. If the particle were to move inward, perhaps down a deep mine shaft, the gravitational force would change for two reasons. (1) It would tend to increase because the particle would
be moving closer to the center of Earth. (2) It would tend to decrease because the thickening shell of material lying outside the particle's radial position would not exert any net force on the particle.

For a uniform Earth, the second influence would prevail and the force on the particle would steadily decrease to zero as the particle approached the center of Earth. However, for the real (nonuniform) Earth, the force on the particle actually increases as the particle begins to descend. The force reaches a maximum at a certain depth and then decreases as the particle descends farther.

**Tunnel through Earth's center, gravitation**

In *Pole to Pole*, an early science fiction story by George Griffith, three explorers attempt to travel by capsule through a naturally formed (and, of course, fictional) tunnel directly from the south pole to the north pole (Fig. 13-7). According to the story, as the capsule approaches Earth's center, the gravitational force on the explorers becomes alarmingly large and then, exactly at the center, it suddenly but only momentarily disappears. Then the capsule travels through the second half of the tunnel, to the north pole.

![Figure 13-7](image)

A capsule of mass \( m \) falls through a tunnel that connects Earth's south and north poles. When the capsule is at distance \( r \) from Earth's center, the portion of Earth's mass that is contained in a sphere of that radius is \( M_{\text{ins}} \).

Check Griffith's description by finding the gravitational force on the capsule of mass \( m \) when it reaches a distance \( r \) from Earth's center. Assume that Earth is a sphere of uniform density \( \rho \) (mass per unit volume).

**KEY IDEAS**

Newton's shell theorem gives us three ideas:

1. When the capsule is at radius \( r \) from Earth's center, the portion of Earth that lies outside a sphere of radius \( r \) does not produce a net gravitational force on the capsule.
2. The portion of Earth that lies inside that sphere does produce a net gravitational force on the capsule.
3. We can treat the mass \( M_{\text{ins}} \) of that inside portion of Earth as being the mass of a particle located at Earth's center.

**Calculations:**

All three ideas tell us that we can write Eq. 13-1, for the magnitude of the gravitational force on the capsule, as

\[
F = \frac{GmM_{\text{ins}}}{r^2}
\]  

(13-17)
To write the mass $M_{\text{ins}}$ in terms of the radius $r$, we note that the volume $V_{\text{ins}}$ containing this mass is $\frac{4}{3}\pi r^3$. Also, because we're assuming an Earth of uniform density, the density $\rho_{\text{ins}} = \frac{M_{\text{ins}}}{V_{\text{ins}}}$ is Earth's density $\rho$. Thus, we have

$$M_{\text{ins}} = \rho V_{\text{ins}} = \rho \frac{4}{3}\pi r^3.$$  \hspace{1cm} (13-18)

Then, after substituting this expression into Eq. 13-17 and canceling, we have

$$F = \frac{4\pi G m \rho}{3} r.$$ \hspace{1cm} \text{(Answer)} \hspace{1cm} (13-19)

This equation tells us that the force magnitude $F$ depends linearly on the capsule's distance $r$ from Earth's center. Thus, as $r$ decreases, $F$ also decreases (opposite of Griffith's description), until it is zero at Earth's center. At least Griffith got the zero-at-the-center detail correct.

Equation 13-19 can also be written in terms of the force vector $\vec{F}$ and the capsule's position vector $\vec{r}$ along a radial axis extending from Earth's center. Let $K$ represent the collection of constants $4\pi G m \rho/3$. Then, Eq. 13-19 becomes

$$\vec{F} = -K \vec{r},$$ \hspace{1cm} (13-20)

in which we have inserted a minus sign to indicate that $\vec{F}$ and $\vec{r}$ have opposite directions. Equation 13-20 has the form of Hooke's law (Eq. 7-20, $\vec{F} = -k \vec{d}$). Thus, under the idealized conditions of the story, the capsule would oscillate like a block on a spring, with the center of the oscillation at Earth's center. After the capsule had fallen from the south pole to Earth's center, it would travel from the center to the north pole (as Griffith said) and then back again, repeating the cycle forever.

---

13-6 Gravitational Potential Energy

In Section 8-4, we discussed the gravitational potential energy of a particle–Earth system. We were careful to keep the particle near Earth's surface, so that we could regard the gravitational force as constant. We then chose some reference configuration of the system as having a gravitational potential energy of zero. Often, in this configuration the particle was on Earth's surface. For particles not on Earth's surface, the gravitational potential energy decreased when the separation between the particle and Earth decreased.

Here, we broaden our view and consider the gravitational potential energy $U$ of two particles, of masses $m$ and $M$, separated by a distance $r$. We again choose a reference configuration with $U$ equal to zero. However, to simplify the equations, the separation distance $r$ in the reference configuration is now large enough to be approximated as infinite. As before, the gravitational potential energy decreases when the separation decreases. Since $U = 0$ for $r = \infty$, the potential energy is negative for any finite separation and becomes progressively more negative as the particles move closer together.

With these facts in mind and as we shall justify next, we take the gravitational potential energy of the two-particle system to be

$$U = -\frac{GMm}{r} \hspace{1cm} \text{(gravitational potential energy)}.$$ \hspace{1cm} (13-21)

Note that $U(r)$ approaches zero as $r$ approaches infinity and that for any finite value of $r$, the value of $U(r)$ is negative.
The potential energy given by Eq. 13-21 is a property of the system of two particles rather than of either particle alone. There is no way to divide this energy and say that so much belongs to one particle and so much to the other. However, if $M \gg m$, as is true for Earth (mass $M$) and a baseball (mass $m$), we often speak of “the potential energy of the baseball.” We can get away with this because, when a baseball moves in the vicinity of Earth, changes in the potential energy of the baseball–Earth system appear almost entirely as changes in the kinetic energy of the baseball, since changes in the kinetic energy of Earth are too small to be measured. Similarly, in Section 13-8 we shall speak of “the potential energy of an artificial satellite” orbiting Earth, because the satellite’s mass is so much smaller than Earth’s mass. When we speak of the potential energy of bodies of comparable mass, however, we have to be careful to treat them as a system.

If our system contains more than two particles, we consider each pair of particles in turn, calculate the gravitational potential energy of that pair with Eq. 13-21 as if the other particles were not there, and then algebraically sum the results. Applying Eq. 13-21 to each of the three pairs of Fig. 13-8 gives the potential energy of the system as

$$U = \left( \frac{Gm_1 m_2}{r_{12}} + \frac{Gm_1 m_3}{r_{13}} + \frac{Gm_2 m_3}{r_{23}} \right)$$

(13-22)

![Figure 13-8](image)

**Figure 13-8** A system consisting of three particles. The gravitational potential energy of the system is the sum of the gravitational potential energies of all three pairs of particles.

**Proof of Equation 13-21**

Let us shoot a baseball directly away from Earth along the path in Fig. 13-9. We want to find an expression for the gravitational potential energy $U$ of the ball at point $P$ along its path, at radial distance $R$ from Earth's center. To do so, we first find the work $W$ done on the ball by the gravitational force as the ball travels from point $P$ to a great (infinite) distance from Earth. Because the gravitational force $\mathbf{F}(r)$ is a variable force (its magnitude depends on $r$), we must use the techniques of Section 7-8 to find the work. In vector notation, we can write

$$W = \int_{R}^{\infty} \mathbf{F}(r) \cdot d\mathbf{r}.$$

(13-23)
A baseball is shot directly away from Earth, through point $P$ at radial distance $R$ from Earth's center. The gravitational force $\vec{F}$ on the ball and a differential displacement vector $d\vec{r}$ are shown, both directed along a radial $r$ axis.

The integral contains the scalar (or dot) product of the force $\vec{F}(r)$ and the differential displacement vector $d\vec{r}$ along the ball's path. We can expand that product as

$$\vec{F}(r) \cdot d\vec{r} = F(r)dr \cos \phi,$$  \hspace{1cm} (13-24)

where $\phi$ is the angle between the directions of $\vec{F}(r)$ and $d\vec{r}$. When we substitute $180^\circ$ for $\phi$ and Eq. 13-1 for $F(r)$, Eq. 13-24 becomes

$$\vec{F}(r) \cdot d\vec{r} = -\frac{GMm}{r^2}dr,$$

where $M$ is Earth's mass and $m$ is the mass of the ball.

Substituting this into Eq. 13-23 and integrating give us

$$W = -GMm\int_{R}^{\infty} \frac{1}{r^2} dr = -\left[\frac{GMm}{r}\right]_{R}^{\infty} = 0 - \frac{GMm}{R} = -\frac{GMm}{R},$$  \hspace{1cm} (13-25)

where $W$ is the work required to move the ball from point $P$ (at distance $R$) to infinity. Equation 8-1 ($\Delta U = -W$) tells us that we can also write that work in terms of potential energies as

$$U_\infty - U = -W.$$  

Because the potential energy $U_\infty$ at infinity is zero, $U$ is the potential energy at $P$, and $W$ is given by Eq. 13-25, this equation becomes

$$U = W = -\frac{GMm}{R}.$$  

Switching $R$ to $r$ gives us Eq. 13-21, which we set out to prove.
Path Independence

In Fig. 13-10, we move a baseball from point $A$ to point $G$ along a path consisting of three radial lengths and three circular arcs (centered on Earth). We are interested in the total work $W$ done by Earth’s gravitational force $\vec{F}$ on the ball as it moves from $A$ to $G$. The work done along each circular arc is zero, because the direction of $\vec{F}$ is perpendicular to the arc at every point. Thus, $W$ is the sum of only the works done by $\vec{F}$ along the three radial lengths.

Now, suppose we mentally shrink the arcs to zero. We would then be moving the ball directly from $A$ to $G$ along a single radial length. Does that change $W$? No. Because no work was done along the arcs, eliminating them does not change the work. The path taken from $A$ to $G$ now is clearly different, but the work done by $\vec{F}$ is the same.

We discussed such a result in a general way in Section 8-3. Here is the point: The gravitational force is a conservative force. Thus, the work done by the gravitational force on a particle moving from an initial point $i$ to a final point $f$ is independent of the path taken between the points. From Eq. 8-1, the change $\Delta U$ in the gravitational potential energy from point $i$ to point $f$ is given by

$$\Delta U = U_f - U_i = -W.$$  \hspace{1cm} (13-26)

Since the work $W$ done by a conservative force is independent of the actual path taken, the change $\Delta U$ in gravitational potential energy is also independent of the path taken.

Potential Energy and Force

In the proof of Eq. 13-21, we derived the potential energy function $U(r)$ from the force function $\vec{F}(r)$. We should be able to go the other way—that is, to start from the potential energy function and derive the force function. Guided by Eq. 8-22 ($F(x) = -dU(x)/dx$), we can write
This is Newton's law of gravitation (Eq. 13-1). The minus sign indicates that the force on mass \( m \) points radially inward, toward mass \( M \).

### Escape Speed

If you fire a projectile upward, usually it will slow, stop momentarily, and return to Earth. There is, however, a certain minimum initial speed that will cause it to move upward forever, theoretically coming to rest only at infinity. This minimum initial speed is called the (Earth) **escape speed**.

Consider a projectile of mass \( m \), leaving the surface of a planet (or some other astronomical body or system) with escape speed \( v \). The projectile has a kinetic energy \( K \) given by \( \frac{1}{2}mv^2 \) and a potential energy \( U \) given by Eq. 13-21:

\[
U = -\frac{GMm}{R},
\]

in which \( M \) is the mass of the planet and \( R \) is its radius.

When the projectile reaches infinity, it stops and thus has no kinetic energy. It also has no potential energy because an infinite separation between two bodies is our zero-potential-energy configuration. Its total energy at infinity is therefore zero. From the principle of conservation of energy, its total energy at the planet's surface must also have been zero, and so

\[
K + U = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0.
\]

This yields

\[
v = \sqrt{\frac{2GM}{R}}.
\]

Note that \( v \) does not depend on the direction in which a projectile is fired from a planet. However, attaining that speed is easier if the projectile is fired in the direction the launch site is moving as the planet rotates about its axis. For example, rockets are launched eastward at Cape Canaveral to take advantage of the Cape's eastward speed of 1500 km/h due to Earth's rotation.

Equation 13-28 can be applied to find the escape speed of a projectile from any astronomical body, provided we substitute the mass of the body for \( M \) and the radius of the body for \( R \). Table 13-2 shows some escape speeds.

<table>
<thead>
<tr>
<th>Body</th>
<th>Mass (kg)</th>
<th>Radius (m)</th>
<th>Escape Speed (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceres(^a)</td>
<td>(1.17 \times 10^{21})</td>
<td>(3.8 \times 10^5)</td>
<td>0.64</td>
</tr>
<tr>
<td>Earth's moon(^b)</td>
<td>(7.36 \times 10^{22})</td>
<td>(1.74 \times 10^6)</td>
<td>2.38</td>
</tr>
<tr>
<td>Earth</td>
<td>(5.98 \times 10^{24})</td>
<td>(6.37 \times 10^6)</td>
<td>11.2</td>
</tr>
<tr>
<td>Jupiter</td>
<td>(1.90 \times 10^{30})</td>
<td>(7.15 \times 10^7)</td>
<td>59.5</td>
</tr>
<tr>
<td>Sun</td>
<td>(1.99 \times 10^{30})</td>
<td>(6.96 \times 10^8)</td>
<td>618</td>
</tr>
<tr>
<td>Sirius B(^b)</td>
<td>(2 \times 10^{30})</td>
<td>(1 \times 10^7)</td>
<td>5200</td>
</tr>
<tr>
<td>Neutron star(^c)</td>
<td>(2 \times 10^{30})</td>
<td>(1 \times 10^4)</td>
<td>(2 \times 10^5)</td>
</tr>
</tbody>
</table>

\(^a\) Mass of Ceres is estimated.

\(^b\) Average for the moon's gravitational field.

\(^c\) Neutron star mass is estimated.
You move a ball of mass $m$ away from a sphere of mass $M$. (a) Does the gravitational potential energy of the system of ball and sphere increase or decrease? (b) Is positive work or negative work done by the gravitational force between the ball and the sphere?

Asteroid falling from space, mechanical energy

An asteroid, headed directly toward Earth, has a speed of 12 km/s relative to the planet when the asteroid is 10 Earth radii from Earth's center. Neglecting the effects of Earth's atmosphere on the asteroid, find the asteroid's speed $v_f$ when it reaches Earth's surface.

Because we are to neglect the effects of the atmosphere on the asteroid, the mechanical energy of the asteroid–Earth system is conserved during the fall. Thus, the final mechanical energy (when the asteroid reaches Earth's surface) is equal to the initial mechanical energy. With kinetic energy $K$ and gravitational potential energy $U$, we can write this as

$$K_f + U_f = K_i + U_i.$$  \hspace{1cm} (13-29)

Also, if we assume the system is isolated, the system's linear momentum must be conserved during the fall. Therefore, the momentum change of the asteroid and that of Earth must be equal in magnitude and opposite in sign. However, because Earth's mass is so much greater than the asteroid's mass, the change in Earth's speed is negligible relative to the change in the asteroid's speed. So, the change in Earth's kinetic energy is also negligible. Thus, we can assume that the kinetic energies in Eq. 13-29 are those of the asteroid alone.

Calculations:

Let $m$ represent the asteroid's mass and $M$ represent Earth's mass ($5.98 \times 10^{24}$ kg). The asteroid is initially at distance $10R_E$ and finally at distance $R_E$, where $R_E$ is Earth's radius ($6.37 \times 10^6$ m). Substituting Eq. 13-21 for $U$ and $\frac{1}{2}mv_i^2$ for $K$, we rewrite Eq. 13-29 as

$$\frac{1}{2}mv_f^2 - \frac{GMm}{R_E} = \frac{1}{2}mv_i^2 - \frac{GMm}{10R_E}.$$

Rearranging and substituting known values, we find

$$v_f^2 = v_i^2 + \frac{2GM}{R_E} \left(1 - \frac{1}{10}\right)$$

$$= \left(12 \times 10^3 \text{ m} / \text{s}\right)^2 + \frac{2\left(6.67 \times 10^{-11} \text{ m}^3 / \text{kg} \cdot \text{s}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)\left(0.9\right)}{6.37 \times 10^6 \text{ m}}$$

$$= 2.557 \times 10^8 \text{ m}^2 / \text{s}^2,$$

and
At this speed, the asteroid would not have to be particularly large to do considerable damage at impact. If it were only 5 m across, the impact could release about as much energy as the nuclear explosion at Hiroshima. Alarmingly, about 500 million asteroids of this size are near Earth’s orbit, and in 1994 one of them apparently penetrated Earth’s atmosphere and exploded 20 km above the South Pacific (setting off nuclear-explosion warnings on six military satellites). The impact of an asteroid 500 m across (there may be a million of them near Earth’s orbit) could end modern civilization and almost eliminate humans worldwide.

13-7 Planets and Satellites: Kepler's Laws

The motions of the planets, as they seemingly wander against the background of the stars, have been a puzzle since the dawn of history. The “loop-the-loop” motion of Mars, shown in Fig. 13-11, was particularly baffling. Johannes Kepler (1571–1630), after a lifetime of study, worked out the empirical laws that govern these motions. Tycho Brahe (1546–1601), the last of the great astronomers to make observations without the help of a telescope, compiled the extensive data from which Kepler was able to derive the three laws of planetary motion that now bear Kepler’s name. Later, Newton (1642–1727) showed that his law of gravitation leads to Kepler’s laws.

Figure 13-11 The path seen from Earth for the planet Mars as it moved against a background of the constellation Capricorn during 1971. The planet’s position on four days is marked. Both Mars and Earth are moving in orbits around the Sun so that we see the position of Mars relative to us; this relative motion sometimes results in an apparent loop in the path of Mars.

In this section we discuss each of Kepler’s three laws. Although here we apply the laws to planets orbiting the Sun, they hold equally well for satellites, either natural or artificial, orbiting Earth or any other massive central body.

1. The Law Of Orbits. All planets move in elliptical orbits, with the Sun at one focus.

Figure 13-12 shows a planet of mass \( m \) moving in such an orbit around the Sun, whose mass is \( M \). We assume that \( M \gg m \), so that the center of mass of the planet–Sun system is approximately at the center of the Sun.
A planet of mass $m$ moving in an elliptical orbit around the Sun. The Sun, of mass $M$, is at one focus $F$ of the ellipse. The other focus is $F'$, which is located in empty space. Each focus is a distance $ea$ from the ellipse’s center, with $e$ being the eccentricity of the ellipse. The semimajor axis $a$ of the ellipse, the perihelion (nearest the Sun) distance $R_p$, and the aphelion (farthest from the Sun) distance $R_a$ are also shown.

The orbit in Fig. 13-12 is described by giving its **semimajor axis** $a$ and its **eccentricity** $e$, the latter defined so that $ea$ is the distance from the center of the ellipse to either focus $F$ or $F'$. An eccentricity of zero corresponds to a circle, in which the two foci merge to a single central point. The eccentricities of the planetary orbits are not large; so if the orbits are drawn to scale, they look circular. The eccentricity of the ellipse of Fig. 13-12, which has been exaggerated for clarity, is 0.74. The eccentricity of Earth’s orbit is only 0.0167.

**2. The Law Of Areas.** A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet’s orbit in equal time intervals; that is, the rate $dA/dt$ at which it sweeps out area $A$ is constant.

Qualitatively, this second law tells us that the planet will move most slowly when it is farthest from the Sun and most rapidly when it is nearest to the Sun. As it turns out, Kepler’s second law is totally equivalent to the law of conservation of angular momentum. Let us prove it.

The area of the shaded wedge in Fig. 13-13a closely approximates the area swept out in time $\Delta t$ by a line connecting the Sun and the planet, which are separated by distance $r$. The area $\Delta A$ of the wedge is approximately the area of a triangle with base $rA\theta$ and height $r$. Since the area of a triangle is one-half of the base times the height, $\Delta A \approx \frac{1}{2} r^2 \Delta \theta$. This expression for $\Delta A$ becomes more exact as $\Delta t$ (hence $\Delta \theta$) approaches zero. The instantaneous rate at which area is being swept out is then

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega, \quad (13-30)$$
The planet sweeps out this area. These are the two momentum components.

Figure 13-13 (a) In time $\Delta t$, the line $r$ connecting the planet to the Sun moves through an angle $\Delta \theta$, sweeping out an area $\Delta A$ (shaded). (b) The linear momentum $\vec{P}$ of the planet and the components of $\vec{P}$.

in which $\omega$ is the angular speed of the rotating line connecting Sun and planet.

Figure 13-13b shows the linear momentum $\vec{P}$ of the planet, along with the radial and perpendicular components of $\vec{P}$. From Eq. 11-20 ($L = rp_\perp$), the magnitude of the angular momentum $\vec{L}$ of the planet about the Sun is given by the product of $r$ and $p_\perp$, the component of $\vec{P}$ perpendicular to $r$. Here, for a planet of mass $m$,

$$L = rp_\perp = (r)(mv_\perp) = (r)(m\omega r)$$

$$= mr^2\omega,$$  \hspace{1cm} (13-31)

where we have replaced $v_\perp$ with its equivalent $\omega r$ (Eq. 10-18). Eliminating $r^2\omega$ between Eqs. 13-30 and 13-31 leads to

$$\frac{dA}{dt} = \frac{L}{2m}.$$  \hspace{1cm} (13-32)

If $dA/dt$ is constant, as Kepler said it is, then Eq. 13-32 means that $L$ must also be constant—angular momentum is conserved. Kepler's second law is indeed equivalent to the law of conservation of angular momentum.

3. The Law Of Periods. The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

To see this, consider the circular orbit of Fig. 13-14, with radius $r$ (the radius of a circle is equivalent to the semimajor axis of an ellipse). Applying Newton's second law ($F = ma$) to the orbiting planet in Fig. 13-14 yields

$$\frac{GMm}{r^2} = \left\{m\right\}\left(\omega^2r\right).$$  \hspace{1cm} (13-33)
Figure 13-14 A planet of mass $m$ moving around the Sun in a circular orbit of radius $r$. Here we have substituted from Eq. 13-1 for the force magnitude $F$ and used Eq. 10-23 to substitute $\omega^2 r$ for the centripetal acceleration. If we now use Eq. 10-20 to replace $\omega$ with $2\pi/T$, where $T$ is the period of the motion, we obtain Kepler’s third law:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \quad \text{(law of periods)}$$

(13-34)

The quantity in parentheses is a constant that depends only on the mass $M$ of the central body about which the planet orbits.

Equation 13-34 holds also for elliptical orbits, provided we replace $r$ with $a$, the semimajor axis of the ellipse. This law predicts that the ratio $T^2/a^3$ has essentially the same value for every planetary orbit around a given massive body. Table 13-3 shows how well it holds for the orbits of the planets of the solar system.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Semimajor Axis $a$ ($10^{10}$ m)</th>
<th>Period $T$ (y)</th>
<th>$T^2/a^3$ ($10^{-34}$ y$^2$/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>5.79</td>
<td>0.241</td>
<td>2.99</td>
</tr>
<tr>
<td>Venus</td>
<td>10.8</td>
<td>0.615</td>
<td>3.00</td>
</tr>
<tr>
<td>Earth</td>
<td>15.0</td>
<td>1.00</td>
<td>2.96</td>
</tr>
<tr>
<td>Mars</td>
<td>22.8</td>
<td>1.88</td>
<td>2.98</td>
</tr>
<tr>
<td>Jupiter</td>
<td>77.8</td>
<td>11.9</td>
<td>3.01</td>
</tr>
<tr>
<td>Saturn</td>
<td>143</td>
<td>29.5</td>
<td>2.98</td>
</tr>
<tr>
<td>Uranus</td>
<td>287</td>
<td>84.0</td>
<td>2.98</td>
</tr>
<tr>
<td>Neptune</td>
<td>450</td>
<td>165</td>
<td>2.99</td>
</tr>
<tr>
<td>Pluto</td>
<td>590</td>
<td>248</td>
<td>2.99</td>
</tr>
</tbody>
</table>

CHECKPOINT 4

Satellite 1 is in a certain circular orbit around a planet, while satellite 2 is in a larger circular orbit. Which satellite has (a) the longer period and (b) the greater speed?
Kepler's law of periods, Comet Halley

Comet Halley orbits the Sun with a period of 76 years and, in 1986, had a distance of closest approach to the Sun, its *perihelion distance* \( R_p \), of \( 8.9 \times 10^{10} \) m. Table 13-3 shows that this is between the orbits of Mercury and Venus.

(a) What is the comet’s farthest distance from the Sun, which is called its *aphelion distance* \( R_a \)?

**KEY IDEAS**

From Fig. 13-12, we see that \( R_a + R_p = 2a \), where \( a \) is the semi-major axis of the orbit. Thus, we can find \( R_a \) if we first find \( a \). We can relate \( a \) to the given period via the law of periods (Eq. 13-34) if we simply substitute the semimajor axis \( a \) for \( r \).

**Calculations:**

Making that substitution and then solving for \( a \), we have

\[
a = \left( \frac{GM^2}{4\pi^2} \right)^{1/3}.
\]

(13-35)

If we substitute the mass \( M \) of the Sun, \( 1.99 \times 10^{30} \) kg, and the period \( T \) of the comet, 76 years or \( 2.4 \times 10^9 \) s, into Eq. 13-35, we find that \( a = 2.7 \times 10^{12} \) m. Now we have

\[
R_a = 2a - R_p
\]

\[
= \left( 2 \right) \left( 2.7 \times 10^{12} \text{ m} \right) - 8.9 \times 10^{10} \text{ m}
\]

(Answer)

\[
= 5.3 \times 10^{12} \text{ m}
\]

Table 13-3 shows that this is a little less than the semimajor axis of the orbit of Pluto. Thus, the comet does not get farther from the Sun than Pluto.

(b) What is the eccentricity \( e \) of the orbit of comet Halley?

**KEY IDEAS**

We can relate \( e, a, \) and \( R_p \) via Fig. 13-12, in which we see that \( ea = a - R_p \).

**Calculations:**

We have

\[
e = \frac{a - R_p}{a} = 1 - \frac{R_p}{a}
\]

(13-36)

\[
e = 1 - \frac{8.9 \times 10^{10} \text{ m}}{2.7 \times 10^{12} \text{ m}} = 0.97.
\]

(Answer)

This tells us that, with an eccentricity approaching unity, this orbit must be a long thin ellipse.
As a satellite orbits Earth in an elliptical path, both its speed, which fixes its kinetic energy $K$, and its distance from the center of Earth, which fixes its gravitational potential energy $U$, fluctuate with fixed periods. However, the mechanical energy $E$ of the satellite remains constant. (Since the satellite's mass is so much smaller than Earth's mass, we assign $U$ and $E$ for the Earth–satellite system to the satellite alone.)

The potential energy of the system is given by Eq. 13-21:

$$U = -\frac{GMm}{r}$$

(with $U = 0$ for infinite separation). Here $r$ is the radius of the satellite's orbit, assumed for the time being to be circular, and $M$ and $m$ are the masses of Earth and the satellite, respectively.

To find the kinetic energy of a satellite in a circular orbit, we write Newton's second law ($F = ma$) as

$$\frac{GMm}{r^2} = m\frac{v^2}{r}.$$

(13-37)

where $v^2/r$ is the centripetal acceleration of the satellite. Then, from Eq. 13-37, the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r},$$

(13-38)

which shows us that for a satellite in a circular orbit,

$$K = -\frac{U}{2} \text{ (circular orbit).}$$

(13-39)

The total mechanical energy of the orbiting satellite is

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r}$$

or

$$E = -\frac{GMm}{2r} \text{ (circular orbit).}$$

(13-40)

This tells us that for a satellite in a circular orbit, the total energy $E$ is the negative of the kinetic energy $K$:

$$E = -K \text{ (circular orbit).}$$

(13-41)

For a satellite in an elliptical orbit of semimajor axis $a$, we can substitute $a$ for $r$ in Eq. 13-40 to find the mechanical energy:

$$E = -\frac{GMm}{2a} \text{ (elliptical orbit)}$$

(13-42)

Equation 13-42 tells us that the total energy of an orbiting satellite depends only on the semimajor axis of its orbit and not on its eccentricity $e$. For example, four orbits with the same semimajor axis are shown in Fig. 13-15; the same satellite would have the same total mechanical energy $E$ in all four orbits. Figure 13-16 shows the variation of $K$, $U$, and $E$ with $r$ for a satellite moving in a circular orbit about a massive central body.
Figure 13-15 Four orbits with different eccentricities $e$ about an object of mass $M$. All four orbits have the same semimajor axis $a$ and thus correspond to the same total mechanical energy $E$.

This is a plot of a satellite's energies versus orbit radius.

Figure 13-16 The variation of kinetic energy $K$, potential energy $U$, and total energy $E$ with radius $r$ for a satellite in a circular orbit. For any value of $r$, the values of $U$ and $E$ are negative, the value of $K$ is positive, and $E = -K$. As $r \to \infty$, all three energy curves approach a value of zero.

CHECKPOINT 5

In the figure here, a space shuttle is initially in a circular orbit of radius $r$ about Earth. At point $P$, the pilot briefly fires a forward-pointing thruster to decrease the shuttle's kinetic energy $K$ and mechanical energy $E$. (a) Which of the dashed elliptical orbits shown in the figure will the shuttle then take? (b) Is the orbital period $T$ of the shuttle (the time to return to $P$) then greater than, less than, or the same as in the circular orbit?
Mechanical energy of orbiting bowling ball

A playful astronaut releases a bowling ball, of mass \( m = 7.20 \text{ kg} \), into circular orbit about Earth at an altitude \( h \) of 350 km.

(d) What is the mechanical energy \( E \) of the ball in its orbit?

**KEY IDEAS**

We can get \( E \) from the orbital energy, given by Eq. 13-40 \((E = -\frac{GMm}{2r})\), if we first find the orbital radius \( r \). (It is not simply the given altitude.)

**Calculations:**

The orbital radius must be

\[
r = R + h = 6370 \text{ km} + 350 \text{ km} = 6.72 \times 10^6 \text{ m},
\]

in which \( R \) is the radius of Earth. Then, from Eq. 13-40, the mechanical energy is

\[
E = -\frac{GMm}{2r} = -\frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)(7.20 \text{ kg})}{2\left(6.72 \times 10^6 \text{ m}\right)}
\]

\[
= -2.14 \times 10^8 \text{ J} = -214 \text{ MJ}
\]

(e) What is the mechanical energy \( E_0 \) of the ball on the launchpad at Cape Canaveral (before it, the astronaut, and the spacecraft are launched)? From there to the orbit, what is the change \( \Delta E \) in the ball’s mechanical energy?

**KEY IDEAS**

On the launchpad, the ball is *not* in orbit and thus Eq. 13-40 does *not* apply. Instead, we must find \( E_0 = K_0 + U_0 \), where \( K_0 \) is the ball’s kinetic energy and \( U_0 \) is the gravitational potential energy of the ball–Earth system.

**Calculations:**
To find \( U_0 \), we use Eq. 13-21 to write
\[
U_0 = - \frac{GMm}{R} = - \left( \frac{6.67 \times 10^{-11} \text{ N m}^2 / \text{ kg}^2}{6.37 \times 10^6 \text{ m}} \right) \left( 5.98 \times 10^{24} \text{ kg} \right) (7.2 \text{ kg})
\]
\[
= -4.51 \times 10^8 \text{ J} = -451 \text{ MJ}
\]

The kinetic energy \( K_0 \) of the ball is due to the ball's motion with Earth's rotation. You can show that \( K_0 \) is less than 1 MJ, which is negligible relative to \( U_0 \). Thus, the mechanical energy of the ball on the launchpad is
\[
E_0 = K_0 + U_0 \approx 0 - 451 \text{ MJ} = -451 \text{ MJ.}
\]

The increase in the mechanical energy of the ball from launchpad to orbit is
\[
\Delta E = E - E_0 = (-214 \text{ MJ}) - (-451 \text{ MJ}) = 237 \text{ MJ}
\]

This is worth a few dollars at your utility company. Obviously the high cost of placing objects into orbit is not due to their required mechanical energy.

---

13-9  \hspace{1cm} Einstein and Gravitation

**Principle of Equivalence**

Albert Einstein once said: “I was … in the patent office at Bern when all of a sudden a thought occurred to me: ‘If a person falls freely, he will not feel his own weight.’ I was startled. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation.”

Thus Einstein tells us how he began to form his general theory of relativity. The fundamental postulate of this theory about gravitation (the gravitating of objects toward each other) is called the principle of equivalence, which says that gravitation and acceleration are equivalent. If a physicist were locked up in a small box as in Fig. 13-17, he would not be able to tell whether the box was at rest on Earth (and subject only to Earth’s gravitational force), as in Fig. 13-17a, or accelerating through interstellar space at 9.8 m/s² (and subject only to the force producing that acceleration), as in Fig. 13-17b. In both situations he would feel the same and would read the same value for his weight on a scale. Moreover, if he watched an object fall past him, the object would have the same acceleration relative to him in both situations.
A physicist in a box resting on Earth sees a cantaloupe falling with acceleration \( a = 9.8 \text{ m/s}^2 \). (b) If he and the box accelerate in deep space at \( 9.8 \text{ m/s}^2 \), the cantaloupe has the same acceleration relative to him. It is not possible, by doing experiments within the box, for the physicist to tell which situation he is in. For example, the platform scale on which he stands reads the same weight in both situations.

Curvature of Space

We have thus far explained gravitation as due to a force between masses. Einstein showed that, instead, gravitation is due to a curvature of space that is caused by the masses. (As is discussed later in this book, space and time are entangled, so the curvature of which Einstein spoke is really a curvature of spacetime, the combined four dimensions of our universe.)

Picturing how space (such as vacuum) can have curvature is difficult. An analogy might help: Suppose that from orbit we watch a race in which two boats begin on Earth’s equator with a separation of 20 km and head due south (Fig. 13-18a). To the sailors, the boats travel along flat, parallel paths. However, with time the boats draw together until, nearer the south pole, they touch. The sailors in the boats can interpret this drawing together in terms of a force acting on the boats. Looking on from space, however, we can see that the boats draw together simply because of the curvature of Earth’s surface. We can see this because we are viewing the race from “outside” that surface.

Figure 13-18(a) Two objects moving along lines of longitude toward the south pole converge because of the curvature of Earth’s surface. (b) Two objects falling freely near Earth move along lines that converge toward the center of Earth because of the curvature of space near Earth. (c) Far from Earth (and other masses), space is flat and parallel paths remain parallel. Close to Earth, the parallel paths begin to converge because space is curved by Earth’s mass.
Figure 13-18b shows a similar race: Two horizontally separated apples are dropped from the same height above Earth. Although the apples may appear to travel along parallel paths, they actually move toward each other because they both fall toward Earth's center. We can interpret the motion of the apples in terms of the gravitational force on the apples from Earth. We can also interpret the motion in terms of a curvature of the space near Earth, a curvature due to the presence of Earth's mass. This time we cannot see the curvature because we cannot get “outside” the curved space, as we got “outside” the curved Earth in the boat example. However, we can depict the curvature with a drawing like Fig. 13-18c; there the apples would move along a surface that curves toward Earth because of Earth's mass.

When light passes near Earth, the path of the light bends slightly because of the curvature of space there, an effect called **gravitational lensing**. When light passes a more massive structure, like a galaxy or a black hole having large mass, its path can be bent more. If such a massive structure is between us and a quasar (an extremely bright, extremely distant source of light), the light from the quasar can bend around the massive structure and toward us (Fig. 13-19a). Then, because the light seems to be coming to us from a number of slightly different directions in the sky, we see the same quasar in all those different directions. In some situations, the quasars we see blend together to form a giant luminous arc, which is called an *Einstein ring* (Fig. 13-19b).

![Figure 13-19](image-url)

**Figure 13-19(a)** Light from a distant quasar follows curved paths around a galaxy or a large black hole because the mass of the galaxy or black hole has curved the adjacent space. If the light is detected, it appears to have originated along the backward extensions of the final paths (dashed lines). **(b)** The Einstein ring known as MG1131+0456 on the computer screen of a telescope. The source of the light (actually, radio waves, which are a form of invisible light) is far behind the large, unseen galaxy that produces the ring; a portion of the source appears as the two bright spots seen along the ring. *(Courtesy National Radio Astronomy Observatory)*

Should we attribute gravitation to the curvature of spacetime due to the presence of masses or to a force between masses? Or should we attribute it to the actions of a type of fundamental particle called a *graviton*, as conjectured in some modern physics theories? Although our theories about gravitation have been enormously successful in describing everything from falling apples to planetary and stellar motions, we still do not fully understand it on either the cosmological scale or the quantum physics scale.

Copyright © 2011 John Wiley & Sons, Inc. All rights reserved.
The Law of Gravitation

Any particle in the universe attracts any other particle with a gravitational force whose magnitude is

\[ F = G \frac{m_1 m_2}{r^2} \]  \hspace{0.5cm} (Newton's law of gravitation),  

(13-1)

where \( m_1 \) and \( m_2 \) are the masses of the particles, \( r \) is their separation, and \( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \) is the gravitational constant.

Gravitational Behavior of Uniform Spherical Shells

The gravitational force between extended bodies is found by adding (integrating) the individual forces on individual particles within the bodies. However, if either of the bodies is a uniform spherical shell or a spherically symmetric solid, the net gravitational force it exerts on an external object may be computed as if all the mass of the shell or body were located at its center.

Superposition

Gravitational forces obey the principle of superposition; that is, if \( n \) particles interact, the net force \( \vec{F}_{1, \text{net}} \) on a particle labeled particle 1 is the sum of the forces on it from all the other particles taken one at a time:

\[ \vec{F}_{1, \text{net}} = \sum_{i=2}^{n} \vec{F}_{i1}, \]

(13-5)

in which the sum is a vector sum of the forces \( \vec{F}_{i1} \) on particle 1 from particles 2, 3, ..., \( n \). The gravitational force \( \vec{F}_{1i} \) on a particle from an extended body is found by dividing the body into units of differential mass \( dm \), each of which produces a differential force \( d\vec{F} \) on the particle, and then integrating to find the sum of those forces:

\[ \vec{F}_{1i} = \int d\vec{F}. \]

(13-6)

Gravitational Acceleration

The gravitational acceleration \( a_g \) of a particle (of mass \( m \)) is due solely to the gravitational force acting on it. When the particle is at distance \( r \) from the center of a uniform, spherical body of mass \( M \), the magnitude \( F \) of the gravitational force on the particle is given by Eq. 13-1. Thus, by Newton's second law,

\[ F = ma_g, \]

(13-10)

which gives

\[ a_g = \frac{GM}{r^2}. \]

(13-11)

Free-Fall Acceleration and Weight

Because Earth's mass is not distributed uniformly, because the planet is not perfectly spherical, and because it rotates, the actual free-fall acceleration \( \vec{g} \) of a particle near Earth differs slightly from the gravitational acceleration \( \vec{a}_g \), and the particle's weight (equal to \( mg \)) differs from the magnitude of the gravitational force on it (Eq. 13-1).

Gravitation Within a Spherical Shell

A uniform shell of matter exerts no net gravitational force on a particle located inside it. This means that if a particle is located inside a uniform solid sphere at distance \( r \) from its center, the gravitational force exerted on the particle is due only to the mass \( M_{\text{ins}} \) that lies inside a sphere of radius \( r \). This mass is given by

\[ M_{\text{ins}} = \rho V_{\text{ins}} = \rho \frac{4\pi r^3}{3}. \]

(13-18)

where \( \rho \) is the density of the sphere.
Gravitational Potential Energy  The gravitational potential energy $U(r)$ of a system of two particles, with masses $M$ and $m$ and separated by a distance $r$, is the negative of the work that would be done by the gravitational force of either particle acting on the other if the separation between the particles were changed from infinite (very large) to $r$. This energy is

$$U = -\frac{GMm}{r} \quad \text{(gravitational potential energy)}.$$  \hfill (13-21)

Potential Energy of a System  If a system contains more than two particles, its total gravitational potential energy $U$ is the sum of terms representing the potential energies of all the pairs. As an example, for three particles, of masses $m_1$, $m_2$, and $m_3$,

$$U = -\left( \frac{G m_1 m_2}{r_{12}} + \frac{G m_1 m_3}{r_{13}} + \frac{G m_2 m_3}{r_{23}} \right)$$  \hfill (13-22)

Escape Speed  An object will escape the gravitational pull of an astronomical body of mass $M$ and radius $R$ (that is, it will reach an infinite distance) if the object's speed near the body's surface is at least equal to the escape speed, given by

$$v = \sqrt{\frac{2GM}{R}}.$$  \hfill (13-28)

Kepler's Laws  The motion of satellites, both natural and artificial, is governed by these laws:

1. The law of orbits. All planets move in elliptical orbits with the Sun at one focus.
2. The law of areas. A line joining any planet to the Sun sweeps out equal areas in equal time intervals. (This statement is equivalent to conservation of angular momentum.)
3. The law of periods. The square of the period $T$ of any planet is proportional to the cube of the semimajor axis $a$ of its orbit. For circular orbits with radius $r$,

$$T^2 = \left( \frac{4\pi^2}{GM} \right) r^3 \quad \text{(law of periods)},$$  \hfill (13-34)

where $M$ is the mass of the attracting body—the Sun in the case of the solar system. For elliptical planetary orbits, the semimajor axis $a$ is substituted for $r$.

Energy in Planetary Motion  When a planet or satellite with mass $m$ moves in a circular orbit with radius $r$, its potential energy $U$ and kinetic energy $K$ are given by

$$U = -\frac{GMm}{r} \quad \text{and} \quad K = \frac{GMm}{2r}.$$  \hfill (13-21, 13-38)

The mechanical energy $E = K + U$ is then

$$E = -\frac{GMm}{2a}.$$  \hfill (13-40)

For an elliptical orbit of semimajor axis $a$,

$$E = -\frac{GMm}{2a}.$$  \hfill (13-42)

Einstein's View of Gravitation  Einstein pointed out that gravitation and acceleration are equivalent. This principle of equivalence led him to a theory of gravitation (the general theory of relativity) that explains gravitational effects in terms of a curvature of space.
1. In Fig. 13-20, a central particle of mass $M$ is surrounded by a square array of other particles, separated by either distance $d$ or distance $d/2$ along the perimeter of the square. What are the magnitude and direction of the net gravitational force on the central particle due to the other particles?

![Figure 13-20](image)

2. Figure 13-21 shows three arrangements of the same identical particles, with three of them placed on a circle of radius 0.20 m and the fourth one placed at the center of the circle. (a) Rank the arrangements according to the magnitude of the net gravitational force on the central particle due to the other three particles, greatest first. (b) Rank them according to the gravitational potential energy of the four-particle system, least negative first.

![Figure 13-21](image)

3. In Fig. 13-22, a central particle is surrounded by two circular rings of particles, at radii $r$ and $R$, with $R > r$. All the particles have mass $m$. What are the magnitude and direction of the net gravitational force on the central particle due to the particles in the rings?

![Figure 13-22](image)

4. In Fig. 13-23, two particles, of masses $m$ and $2m$, are fixed in place on an axis. (a) Where on the axis can a third particle of mass $3m$ be placed (other than at infinity) so that the net gravitational force on it from the first two particles is zero: to the left of the first two particles, to their right, between them but closer to the more massive particle, or between them but closer to the less massive particle? (b) Does the answer change if the third particle has, instead, a mass of $16m$? (c) Is there a point
off the axis (other than infinity) at which the net force on the third particle would be zero?

Figure 13-23 Question 4.

5 Figure 13-24 shows three situations involving a point particle $P$ with mass $m$ and a spherical shell with a uniformly distributed mass $M$. The radii of the shells are given. Rank the situations according to the magnitude of the gravitational force on particle $P$ due to the shell, greatest first.

Figure 13-24 Question 5.

6 In Fig. 13-25, three particles are fixed in place. The mass of $B$ is greater than the mass of $C$. Can a fourth particle (particle $D$) be placed somewhere so that the net gravitational force on particle $A$ from particles $B$, $C$, and $D$ is zero? If so, in which quadrant should it be placed and which axis should it be near?

Figure 13-25 Question 6.

7 Rank the four systems of equal mass particles shown in Checkpoint 2 according to the absolute value of the gravitational potential energy of the system, greatest first.

Figure 13-26 gives the gravitational acceleration $a_g$ for four planets as a function of the radial distance $r$ from the center of the planet, starting at the surface of the planet (at radius $R_1$, $R_2$, $R_3$, or $R_4$). Plots 1 and 2 coincide for $r \geq R_2$; plots 3 and 4 coincide for $r \geq R_4$. Rank the four planets according to (a) mass and (b) mass per unit volume, greatest first.
9. Figure 13-27 shows three particles initially fixed in place, with B and C identical and positioned symmetrically about the y axis, at distance d from A. (a) In what direction is the net gravitational force $\mathbf{F}_{\text{net}}$ on A? (b) If we move C directly away from the origin, does $\mathbf{F}_{\text{net}}$ change in direction? If so, how and what is the limit of the change?

\[ \begin{align*} \text{Figure 13-27} & \quad \text{Question 9.} \\
\end{align*} \]

10. Figure 13-28 shows six paths by which a rocket orbiting a moon might move from point a to point b. Rank the paths according to (a) the corresponding change in the gravitational potential energy of the rocket–moon system and (b) the net work done on the rocket by the gravitational force from the moon, greatest first.

\[ \begin{align*} \text{Figure 13-28} & \quad \text{Question 10.} \\
\end{align*} \]

11. Figure 13-29 shows three uniform spherical planets that are identical in size and mass. The periods of rotation $T$ for the planets are given, and six lettered points are indicated—three points are on the equators of the planets and three points are on the north poles. Rank the points according to the value of the free-fall acceleration $g$ at them, greatest first.

\[ \begin{align*} \text{Figure 13-29} & \quad \text{Question 11.} \\
\end{align*} \]

12. In Fig. 13-30, a particle of mass $m$ (which is not shown) is to be moved from an infinite distance to one of the three possible locations a, b, and c. Two other particles, of masses $m$ and $2m$, are already fixed in place on the axis, as shown. Rank the three possible locations according to the work done by the net gravitational force on the moving particle due to the fixed particles, greatest first.

\[ \begin{align*} \text{Figure 13-30} & \quad \text{Question 12.} \\
\end{align*} \]
sec. 13-2 Newton’s Law of Gravitation

• 1 ILW A mass $M$ is split into two parts, $m$ and $M - m$, which are then separated by a certain distance. What ratio $m/M$ maximizes the magnitude of the gravitational force between the parts?

• 2 Moon effect. Some people believe that the Moon controls their activities. If the Moon moves from being directly on the opposite side of Earth from you to being directly overhead, by what percent does (a) the Moon’s gravitational pull on you increase and (b) your weight (as measured on a scale) decrease? Assume that the Earth–Moon (center-to-center) distance is $3.82 \times 10^8$ m and Earth’s radius is $6.37 \times 10^6$ m.

• 3 SSM What must the separation be between a 5.2 kg particle and a 2.4 kg particle for their gravitational attraction to have a magnitude of $2.3 \times 10^{-12}$ N?

• 4 The Sun and Earth each exert a gravitational force on the Moon. What is the ratio $F_{Sun}/F_{Earth}$ of these two forces? (The average Sun–Moon distance is equal to the Sun–Earth distance.)

sec. 13-3 Gravitation and the Principle of Superposition

• 5 Miniature black holes. Left over from the big-bang beginning of the universe, tiny black holes might still wander through the universe. If one with a mass of $1 \times 10^{11}$ kg (and a radius of only $1 \times 10^{-16}$ m) reached Earth, at what distance from your head would its gravitational pull on you match that of Earth’s?

• 6 In Fig. 13-31, a square of edge length 20.0 cm is formed by four spheres of masses $m_1 = 5.00$ g, $m_2 = 3.00$ g, $m_3 = 1.00$ g, and $m_4 = 5.00$ g. In unit-vector notation, what is the net gravitational force from them on a central sphere with mass $m_5 = 2.50$ g?

• 7 One dimension. In Fig. 13-32, two point particles are fixed on an $x$ axis separated by distance $d$. Particle $A$ has mass $m_A$ and particle $B$ has mass $3.00m_A$. A third particle $C$, of mass $75.0m_A$, is to be placed on the $x$ axis and near particles $A$ and $B$. In terms of distance $d$, at what $x$ coordinate should $C$ be placed so that the net gravitational force on particle $A$ from particles $B$ and $C$ is zero?
In Fig. 13-33, three 5.00 kg spheres are located at distances $d_1 = 0.300$ m and $d_2 = 0.400$ m. What are the (a) magnitude and (b) direction (relative to the positive direction of the $x$ axis) of the net gravitational force on sphere $B$ due to spheres $A$ and $C$?

![Figure 13-33 Problem 8.](image)

We want to position a space probe along a line that extends directly toward the Sun in order to monitor solar flares. How far from Earth's center is the point on the line where the Sun's gravitational pull on the probe balances Earth's pull?

Two dimensions. In Fig. 13-34, three point particles are fixed in place in an $xy$ plane. Particle $A$ has mass $m_A$, particle $B$ has mass $2.00m_A$, and particle $C$ has mass $3.00m_A$. A fourth particle $D$, with mass $4.00m_A$, is to be placed near the other three particles. In terms of distance $d$, at what (a) $x$ coordinate and (b) $y$ coordinate should particle $D$ be placed so that the net gravitational force on particle $A$ from particles $B$, $C$, and $D$ is zero?

![Figure 13-34 Problem 10.](image)

As seen in Fig. 13-35, two spheres of mass $m$ and a third sphere of mass $M$ form an equilateral triangle, and a fourth sphere of mass $m_4$ is at the center of the triangle. The net gravitational force on that central sphere from the three other spheres is zero. (a) What is $M$ in terms of $m$? (b) If we double the value of $m_4$, what then is the magnitude of the net gravitational force on the central sphere?

![Figure 13-35 Problem 11.](image)

In Fig. 13-36a, particle $A$ is fixed in place at $x = -0.20$ m on the $x$ axis and particle $B$, with a mass of 1.0 kg, is fixed in place at the origin. Particle $C$ (not shown) can be moved along the $x$ axis, between particle $B$ and $x = \infty$. Figure 13-36b shows the $x$ component of the net, $\vec{F}_{\text{net},x}$, of the net gravitational force on particle $B$ due to particles $A$ and $C$, as a function of position $x$ of
particle C. The plot actually extends to the right, approaching an asymptote of $-4.17 \times 10^{-10}$ N as $x \to \infty$. What are the masses of (a) particle A and (b) particle C?

![Figure 13-36](image)

**Problem 12.**

**Figure 13-37** shows a spherical hollow inside a lead sphere of radius $R = 4.00$ cm; the surface of the hollow passes through the center of the sphere and “ Touches” the right side of the sphere. The mass of the sphere before hollowing was $M = 2.95$ kg. With what gravitational force does the hollowed-out lead sphere attract a small sphere of mass $m = 0.431$ kg that lies at a distance $d = 9.00$ cm from the center of the lead sphere, on the straight line connecting the centers of the spheres and of the hollow?

![Figure 13-37](image)

**Problem 13.**

**Problem 14.** Three point particles are fixed in position in an $xy$ plane. Two of them, particle A of mass $6.00$ g and particle B of mass $12.0$ g, are shown in Fig. 13-38, with a separation of $d_{AB} = 0.500$ m at angle $\theta = 30^\circ$. Particle C, with mass $8.00$ g, is not shown. The net gravitational force acting on particle A due to particles B and C is $2.77 \times 10^{-14}$ N at an angle of $-163.8^\circ$ from the positive direction of the $x$ axis. What are (a) the $x$ coordinate and (b) the $y$ coordinate of particle C?

![Figure 13-38](image)

**Problem 14.**

**Problem 15.** Three point particles are fixed in place in an $xyz$ coordinate system. Particle A, at the origin, has mass $m_A$. Particle B, at $xyz$ coordinates $(2.00d, 1.00d, 2.00d)$, has mass $2.00m_A$, and particle C, at coordinates $(-1.00d, 2.00d, -3.00d)$, has mass $3.00m_A$. A fourth particle D, with mass $4.00m_A$, is to be placed near the other particles. In terms of distance $d$, at what (a) $x$, (b) $y$, and (c) $z$ coordinate should D be placed so that the net gravitational force on A from B, C, and D is zero?

**Problem 16.** In Fig. 13-39, a particle of mass $m_1 = 0.67$ kg is a distance $d = 23$ cm from one end of a uniform rod with length $L = 3.0$ m and mass $M = 5.0$ kg. What is the magnitude of the gravitational force $\vec{F}$ on the particle from the rod?
sec. 13-4 Gravitation Near Earth's Surface

17 (a) What will an object weigh on the Moon's surface if it weighs 100 N on Earth's surface? (b) How many Earth radii must this same object be from the center of Earth if it is to weigh the same as it does on the Moon?

18 **Mountain pull.** A large mountain can slightly affect the direction of “down” as determined by a plumb line. Assume that we can model a mountain as a sphere of radius \( R = 2.00 \text{ km} \) and density (mass per unit volume) \( 2.6 \times 10^3 \text{ kg/m}^3 \). Assume also that we hang a 0.50 m plumb line at a distance of \( 3R \) from the sphere's center and such that the sphere pulls horizontally on the lower end. How far would the lower end move toward the sphere?

19 **SSM** At what altitude above Earth's surface would the gravitational acceleration be 4.9 m/s\(^2\)?

20 **Mile-high building.** In 1956, Frank Lloyd Wright proposed the construction of a mile-high building in Chicago. Suppose the building had been constructed. Ignoring Earth's rotation, find the change in your weight if you were to ride an elevator from the street level, where you weigh 600 N, to the top of the building.

21 **ILW** Certain neutron stars (extremely dense stars) are believed to be rotating at about 1 rev/s. If such a star has a radius of 20 km, what must be its minimum mass so that material on its surface remains in place during the rapid rotation?

22 The radius \( R_h \) and mass \( M_h \) of a black hole are related by \( R_h = \frac{2GM_h}{c^2} \), where \( c \) is the speed of light. Assume that the gravitational acceleration \( a_g \) of an object at a distance \( r_o = 1.001R_h \) from the center of a black hole is given by Eq. 13-11 (it is, for large black holes). (a) In terms of \( M_h \), find \( a_g \) at \( r_o \). (b) Does \( a_g \) at \( r_o \) increase or decrease as \( M_h \) increases? (c) What is \( a_g \) at \( r_o \) for a very large black hole whose mass is \( 1.55 \times 10^{12} \) times the solar mass of \( 1.99 \times 10^{30} \text{ kg} \)? (d) If an astronaut of height 1.70 m is at \( r_o \) with her feet down, what is the difference in gravitational acceleration between her head and feet? (e) Is the tendency to stretch the astronaut severe?

23 One model for a certain planet has a core of radius \( R \) and mass \( M \) surrounded by an outer shell of inner radius \( R \), outer radius \( 2R \), and mass \( 4M \). If \( M = 4.1 \times 10^{24} \text{ kg} \) and \( R = 6.0 \times 10^6 \text{ m} \), what is the gravitational acceleration of a particle at points (a) \( R \) and (b) \( 3R \) from the center of the planet?

sec. 13-5 Gravitation Inside Earth

24 Two concentric spherical shells with uniformly distributed masses \( M_1 \) and \( M_2 \) are situated as shown in Fig. 13-40. Find the magnitude of the net gravitational force on a particle of mass \( m \), due to the shells, when the particle is located at radial distance (a) \( a \), (b) \( b \), and (c) \( c \).
A solid uniform sphere has a mass of $1.0 \times 10^4$ kg and a radius of 1.0 m. What is the magnitude of the gravitational force due to the sphere on a particle of mass $m$ located at a distance of (a) 1.5 m and (b) 0.50 m from the center of the sphere? (c) Write a general expression for the magnitude of the gravitational force on the particle at a distance $r \leq 1.0$ m from the center of the sphere.

Consider a pulsar, a collapsed star of extremely high density, with a mass $M$ equal to that of the Sun ($1.98 \times 10^{30}$ kg), a radius $R$ of only 12 km, and a rotational period $T$ of 0.041 s. By what percentage does the free-fall acceleration $g$ differ from the gravitational acceleration $a_g$ at the equator of this spherical star?

Figure 13-41 shows, not to scale, a cross section through the interior of Earth. Rather than being uniform throughout, Earth is divided into three zones: an outer crust, a mantle, and an inner core. The dimensions of these zones and the masses contained within them are shown on the figure. Earth has a total mass of $5.98 \times 10^{24}$ kg and a radius of 6370 km. Ignore rotation and assume that Earth is spherical. (a) Calculate $a_g$ at the surface. (b) Suppose that a bore hole (the Mohole) is driven to the crust–mantle interface at a depth of 25.0 km; what would be the value of $a_g$ at the bottom of the hole? (c) Suppose that Earth were a uniform sphere with the same total mass and size. What would be the value of $a_g$ at a depth of 25.0 km? (Precise measurements of $a_g$ are sensitive probes of the interior structure of Earth, although results can be clouded by local variations in mass distribution.)

Assume a planet is a uniform sphere of radius $R$ that (somehow) has a narrow radial tunnel through its center (Fig. 13-7). Also assume we can position an apple anywhere along the tunnel or outside the sphere. Let $F_R$ be the magnitude of the gravitational force on the apple when it is located at the planet’s surface. How far from the surface is there a point where the magnitude is $\frac{1}{2} F_R$ if we move the apple (a) away from the planet and (b) into the tunnel?

sec. 13-6 Gravitational Potential Energy

Figure 13-42 gives the potential energy function $U(r)$ of a projectile, plotted outward from the surface of a planet of radius $R_s$. What least kinetic energy is required of a projectile launched at the surface if the projectile is to “escape” the planet?
•30 In Problem 1, what ratio \( \frac{m}{M} \) gives the least gravitational potential energy for the system?

•31 SSM The mean diameters of Mars and Earth are \( 6.9 \times 10^3 \) km and \( 1.3 \times 10^4 \) km, respectively. The mass of Mars is 0.11 times Earth's mass. (a) What is the ratio of the mean density (mass per unit volume) of Mars to that of Earth? (b) What is the value of the gravitational acceleration on Mars? (c) What is the escape speed on Mars?

•32 (a) What is the gravitational potential energy of the two-particle system in Problem 3? If you triple the separation between the particles, how much work is done (b) by the gravitational force between the particles and (c) by you?

•33 What multiple of the energy needed to escape from Earth gives the energy needed to escape from (a) the Moon and (b) Jupiter?

•34 Figure 13-42 gives the potential energy function \( U(r) \) of a projectile, plotted outward from the surface of a planet of radius \( R_s \). If the projectile is launched radially outward from the surface with a mechanical energy of \(-2.0 \times 10^9 \) J, what are (a) its kinetic energy at radius \( r = 1.25R_s \) and (b) its turning point (see Section 8-6) in terms of \( R_s \)?

•35 Figure 13-43 shows four particles, each of mass 20.0 g, that form a square with an edge length of \( d = 0.600 \) m. If \( d \) is reduced to 0.200 m, what is the change in the gravitational potential energy of the four-particle system?

•36 Zero, a hypothetical planet, has a mass of \( 5.0 \times 10^{23} \) kg, a radius of \( 3.0 \times 10^6 \) m, and no atmosphere. A 10 kg space probe is to be launched vertically from its surface. (a) If the probe is launched with an initial energy of \( 5.0 \times 10^7 \) J, what will be its kinetic energy when it is \( 4.0 \times 10^6 \) m from the center of Zero? (b) If the probe is to achieve a maximum distance of \( 8.0 \times 10^6 \) m from the center of Zero, with what initial kinetic energy must it be launched from the surface of Zero?

•37 The three spheres in Fig. 13-44, with masses \( m_A = 80 \) g, \( m_B = 10 \) g, and \( m_C = 20 \) g, have their centers on a common line, with \( L = 12 \) cm and \( d = 4.0 \) cm. You move sphere B along the line until its center-to-center separation from C is \( d = 4.0 \) cm. How much work is done on sphere B (a) by you and (b) by the net gravitational force on B due to spheres A and C?

•38 In deep space, sphere A of mass 20 kg is located at the origin of an \( x \) axis and sphere B of mass 10 kg is located on the axis at \( x = 0.80 \) m. Sphere B is released from rest while sphere A is held at the origin. (a) What is the gravitational potential energy of the two-sphere system just as B is released? (b) What is the kinetic energy of B when it has moved 0.20 m toward A?

•39 SSM (a) What is the escape speed on a spherical asteroid whose radius is 500 km and whose gravitational acceleration at the surface is \( 3.0 \) m/s\(^2 \)? (b) How far from the surface will a particle go if it leaves the asteroid's surface with a radial speed of 1000 m/s? (c) With what speed will an object hit the asteroid if it is dropped from 1000 km above the surface?

•40 A projectile is shot directly away from Earth's surface. Neglect the rotation of Earth. What multiple of Earth's radius \( R_E \)
gives the radial distance a projectile reaches if (a) its initial speed is 0.500 of the escape speed from Earth and (b) its initial kinetic energy is 0.500 of the kinetic energy required to escape Earth? (c) What is the least initial mechanical energy required at launch if the projectile is to escape Earth?

Two neutron stars are separated by a distance of $1.0 \times 10^{10}$ m. They each have a mass of $1.0 \times 10^{30}$ kg and a radius of $1.0 \times 10^{5}$ m. They are initially at rest with respect to each other. As measured from that rest frame, how fast are they moving when (a) their separation has decreased to one-half its initial value and (b) they are about to collide?

Figure 13-45a shows a particle A that can be moved along a y axis from an infinite distance to the origin. That origin lies at the midpoint between particles B and C, which have identical masses, and the y axis is a perpendicular bisector between them. Distance D is 0.3057 m. Figure 13-45b shows the potential energy $U$ of the three-particle system as a function of the position of particle A along the y axis. The curve actually extends rightward and approaches an asymptote of $-2.7 \times 10^{11}$ J as $y \to \infty$. What are the masses of (a) particles B and C and (b) particle A?

**sec. 13-7 Planets and Satellites: Kepler's Laws**

(a) What linear speed must an Earth satellite have to be in a circular orbit at an altitude of 160 km above Earth's surface? (b) What is the period of revolution?

A satellite is put in a circular orbit about Earth with a radius equal to one-half the radius of the Moon's orbit. What is its period of revolution in lunar months? (A lunar month is the period of revolution of the Moon.)

The Martian satellite Phobos travels in an approximately circular orbit of radius $9.4 \times 10^{6}$ m with a period of 7 h 39 min. Calculate the mass of Mars from this information.

The first known collision between space debris and a functioning satellite occurred in 1996: At an altitude of 700 km, a year-old French spy satellite was hit by a piece of an Ariane rocket. A stabilizing boom on the satellite was demolished, and the satellite was sent spinning out of control. Just before the collision and in kilometers per hour, what was the speed of the rocket piece relative to the satellite if both were in circular orbits and the collision was (a) head-on and (b) along perpendicular paths?

The Sun, which is $2.2 \times 10^{20}$ m from the center of the Milky Way galaxy, revolves around that center once every $2.5 \times 10^{8}$ years. Assuming each star in the Galaxy has a mass equal to the Sun's mass of $2.0 \times 10^{30}$ kg, the stars are distributed uniformly in a sphere about the galactic center, and the Sun is at the edge of that sphere, estimate the number of stars in the Galaxy.

The mean distance of Mars from the Sun is 1.52 times that of Earth from the Sun. From Kepler's law of periods, calculate the number of years required for Mars to make one revolution around the Sun; compare your answer with the value given in Appendix C.

A comet that was seen in April 574 by Chinese astronomers on a day known by them as the Woo Woo day was spotted again in May 1994. Assume the time between observations is the period of the Woo Woo day comet and take its eccentricity as 0.11. What are (a) the semimajor axis of the comet's orbit and (b) its
greatest distance from the Sun in terms of the mean orbital radius \( R_P \) of Pluto?

• **50.** An orbiting satellite stays over a certain spot on the equator of (rotating) Earth. What is the altitude of the orbit (called a geosynchronous orbit)?

• **51.** A satellite, moving in an elliptical orbit, is 360 km above Earth's surface at its farthest point and 180 km above at its closest point. Calculate (a) the semimajor axis and (b) the eccentricity of the orbit.

• **52.** The Sun's center is at one focus of Earth's orbit. How far from this focus is the other focus, (a) in meters and (b) in terms of the solar radius, \( 6.96 \times 10^8 \) m? The eccentricity is 0.0167, and the semimajor axis is \( 1.50 \times 10^{11} \) m.

• **53.** A 20 kg satellite has a circular orbit with a period of 2.4 h and a radius of \( 8.0 \times 10^8 \) m around a planet of unknown mass. If the magnitude of the gravitational acceleration on the surface of the planet is 8.0 m/s\(^2\), what is the radius of the planet?

• **54.** Hunting a black hole. Observations of the light from a certain star indicate that it is part of a binary (two-star) system. This visible star has orbital speed \( v = 270 \) km/s, orbital period \( T = 1.70 \) days, and approximate mass \( m_1 = 6M_s \), where \( M_s \) is the Sun's mass, \( 1.99 \times 10^{30} \) kg. Assume that the visible star and its companion star, which is dark and unseen, are both in circular orbits (Fig. 13-46). What multiple of \( M_s \) gives the approximate mass \( m_2 \) of the dark star?

• **55.** In 1610, Galileo used his telescope to discover four prominent moons around Jupiter. Their mean orbital radii \( a \) and periods \( T \) are as follows:

<table>
<thead>
<tr>
<th>Name</th>
<th>( a ) (10(^8) m)</th>
<th>( T ) (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Io</td>
<td>4.22</td>
<td>1.77</td>
</tr>
<tr>
<td>Europa</td>
<td>6.71</td>
<td>3.55</td>
</tr>
<tr>
<td>Ganymede</td>
<td>10.7</td>
<td>7.16</td>
</tr>
<tr>
<td>Callisto</td>
<td>18.8</td>
<td>16.7</td>
</tr>
</tbody>
</table>

(a) Plot \( \log a \) (y axis) against \( \log T \) (x axis) and show that you get a straight line. (b) Measure the slope of the line and compare it with the value that you expect from Kepler's third law. (c) Find the mass of Jupiter from the intercept of this line with the y axis.

• **56.** In 1993 the spacecraft Galileo sent home an image (Fig. 13-47) of asteroid 243 Ida and a tiny orbiting moon (now known as Dactyl), the first confirmed example of an asteroid–moon system. In the image, the moon, which is 1.5 km wide, is 100 km from the center of the asteroid, which is 55 km long. The shape of the moon's orbit is not well known; assume it is circular with a period of 27 h. (a) What is the mass of the asteroid? (b) The volume of the asteroid, measured from the Galileo images, is 14 100 km\(^3\). What is the density (mass per unit volume) of the asteroid?
Problem 56. A tiny moon (at right) orbits asteroid 243 Ida. (Courtesy NASA)

In a certain binary-star system, each star has the same mass as our Sun, and they revolve about their center of mass. The distance between them is the same as the distance between Earth and the Sun. What is their period of revolution in years?

The presence of an unseen planet orbiting a distant star can sometimes be inferred from the motion of the star as we see it. As the star and planet orbit the center of mass of the star–planet system, the star moves toward and away from us with what is called the line of sight velocity, a motion that can be detected. Figure 13-48 shows a graph of the line of sight velocity versus time for the star 14 Herculis. The star's mass is believed to be 0.90 of the mass of our Sun. Assume that only one planet orbits the star and that our view is along the plane of the orbit. Then approximate (a) the planet’s mass in terms of Jupiter’s mass $m_J$ and (b) the planet’s orbital radius in terms of Earth’s orbital radius $r_E$.

Problem 58.

Three identical stars of mass $M$ form an equilateral triangle that rotates around the triangle’s center as the stars move in a common circle about that center. The triangle has edge length $L$. What is the speed of the stars?

sec. 13-8 Satellites: Orbits and Energy

In Fig. 13-49, two satellites, $A$ and $B$, both of mass $m = 125$ kg, move in the same circular orbit of radius $r = 7.87 \times 10^6$ m around Earth but in opposite senses of rotation and therefore on a collision course. (a) Find the total mechanical energy $E_A + E_B$ of the two satellites + Earth system before the collision. (b) If the collision is completely inelastic so that the wreckage remains as one piece of tangled material (mass = $2m$), find the total mechanical energy immediately after the collision. (c) Just after the collision, is the wreckage falling directly toward Earth’s center or orbiting around Earth?
61. (a) At what height above Earth's surface is the energy required to lift a satellite to that height equal to the kinetic energy required for the satellite to be in orbit at that height? (b) For greater heights, which is greater, the energy for lifting or the kinetic energy for orbiting?

62. Two Earth satellites, A and B, each of mass \( m \), are to be launched into circular orbits about Earth's center. Satellite A is to orbit at an altitude of 6370 km. Satellite B is to orbit at an altitude of 19 110 km. The radius of Earth \( R_E \) is 6370 km. (a) What is the ratio of the potential energy of satellite B to that of satellite A, in orbit? (b) What is the ratio of the kinetic energy of satellite B to that of satellite A, in orbit? (c) Which satellite has the greater total energy if each has a mass of 14.6 kg? (d) By how much?

63. An asteroid, whose mass is \( 2.0 \times 10^{-4} \) times the mass of Earth, revolves in a circular orbit around the Sun at a distance that is twice Earth's distance from the Sun. (a) Calculate the period of revolution of the asteroid in years. (b) What is the ratio of the kinetic energy of the asteroid to the kinetic energy of Earth?

64. A satellite orbits a planet of unknown mass in a circle of radius \( 2.0 \times 10^7 \) m. The magnitude of the gravitational force on the satellite from the planet is \( F = 80 \) N. (a) What is the kinetic energy of the satellite in this orbit? (b) What would \( F \) be if the orbit radius were increased to \( 3.0 \times 10^7 \) m?

65. A satellite is in a circular Earth orbit of radius \( r \). The area \( A \) enclosed by the orbit depends on \( r^2 \) because \( A = \frac{\pi r^2}{2} \). Determine how the following properties of the satellite depend on \( r \): (a) period, (b) kinetic energy, (c) angular momentum, and (d) speed.

66. One way to attack a satellite in Earth orbit is to launch a swarm of pellets in the same orbit as the satellite but in the opposite direction. Suppose a satellite in a circular orbit 500 km above Earth's surface collides with a pellet having mass 4.0 g. (a) What is the kinetic energy of the pellet in the reference frame of the satellite just before the collision? (b) What is the ratio of this kinetic energy to the kinetic energy of a 4.0 g bullet from a modern army rifle with a muzzle speed of 950 m/s?

67. What are (a) the speed and (b) the period of a 220 kg satellite in an approximately circular orbit 640 km above the surface of Earth? Suppose the satellite loses mechanical energy at the average rate of \( 1.4 \times 10^5 \) J per orbital revolution. Adopting the reasonable approximation that the satellite's orbit becomes a “circle of slowly diminishing radius,” determine the satellite's (c) altitude, (d) speed, and (e) period at the end of its 1500th revolution. (f) What is the magnitude of the average retarding force on the satellite? Is angular momentum around Earth's center conserved for (g) the satellite and (h) the satellite–Earth system (assuming that system is isolated)?

68. Two small spaceships, each with mass \( m = 2000 \) kg, are in the circular Earth orbit of Fig. 13-50, at an altitude \( h \) of 400 km. Igor, the commander of one of the ships, arrives at any fixed point in the orbit 90 s ahead of Picard, the commander of the other ship. What are the (a) period \( T_0 \) and (b) speed \( v_0 \) of the ships? At point \( P \) in Fig. 13-50, Picard fires an instantaneous burst in the forward direction, reducing his ship's speed by 1.00%. After this burst, he follows the elliptical orbit shown dashed in the figure. What are the (c) kinetic energy and (d) potential energy of his ship immediately after the burst? In Picard's new elliptical orbit, what are (e) the total energy \( E \), (f) the semimajor axis \( a \), and (g) the orbital period \( T \)? (h) How much earlier than Igor will Picard return to \( P \)?
sec. 13-9 Einstein and Gravitation

69 In Fig. 13.17b, the scale on which the 60 kg physicist stands reads 220 N. How long will the cantaloupe take to reach the floor if the physicist drops it (from rest relative to himself) at a height of 2.1 m above the floor?

Additional Problems

70 The radius \( R_h \) of a black hole is the radius of a mathematical sphere, called the event horizon, that is centered on the black hole. Information from events inside the event horizon cannot reach the outside world. According to Einstein's general theory of relativity, \( R_h = \frac{2GM}{c^2} \), where \( M \) is the mass of the black hole and \( c \) is the speed of light.

Suppose that you wish to study a black hole near it, at a radial distance of 50\( R_h \). However, you do not want the difference in gravitational acceleration between your feet and your head to exceed 10 m/s\(^2\) when you are feet down (or head down) toward the black hole. (a) As a multiple of our Sun's mass \( M_S \), approximately what is the limit to the mass of the black hole you can tolerate at the given radial distance? (You need to estimate your height.) (b) Is the limit an upper limit (you can tolerate smaller masses) or a lower limit (you can tolerate larger masses)?

71 Several planets (Jupiter, Saturn, Uranus) are encircled by rings, perhaps composed of material that failed to form a satellite. In addition, many galaxies contain ring-like structures. Consider a homogeneous thin ring of mass \( M \) and outer radius \( R \) (Fig. 13.51). (a) What gravitational attraction does it exert on a particle of mass \( m \) located on the ring's central axis a distance \( x \) from the ring center? (b) Suppose the particle falls from rest as a result of the attraction of the ring of matter. What is the speed with which it passes through the center of the ring?

72 A typical neutron star may have a mass equal to that of the Sun but a radius of only 10 km. (a) What is the gravitational acceleration at the surface of such a star? (b) How fast would an object be moving if it fell from rest through a distance of 1.0 m on such a star? (Assume the star does not rotate.)

73 Figure 13.52 is a graph of the kinetic energy \( K \) of an asteroid versus its distance \( r \) from Earth's center, as the asteroid falls directly in toward that center. (a) What is the (approximate) mass of the asteroid? (b) What is its speed at \( r = 1.945 \times 10^7 \) m?
The mysterious visitor that appears in the enchanting story *The Little Prince* was said to come from a planet that “was scarcely any larger than a house!” Assume that the mass per unit volume of the planet is about that of Earth and that the planet does not appreciably spin. Approximate (a) the free-fall acceleration on the planet’s surface and (b) the escape speed from the planet.

The masses and coordinates of three spheres are as follows: 20 kg, \(x = 0.50\, \text{m}, y = 1.0\, \text{m}\); 40 kg, \(x = -1.0\, \text{m}, y = -0.50\, \text{m}\). What is the magnitude of the gravitational force on a 20 kg sphere located at the origin due to these three spheres?

A very early, simple satellite consisted of an inflated spherical aluminum balloon 30 m in diameter and of mass 20 kg. Suppose a meteor having a mass of 7.0 kg passes within 3.0 m of the surface of the satellite. What is the magnitude of the gravitational force on the meteor from the satellite at the closest approach?

Four uniform spheres, with masses \(m_A = 40\, \text{kg}, m_B = 35\, \text{kg}, m_C = 200\, \text{kg}, \) and \(m_D = 50\, \text{kg}\), have \((x, y)\) coordinates of \((0, 50\, \text{cm})\), \((0, 0)\), \((-80\, \text{cm}, 0)\), and \((40\, \text{cm}, 0)\), respectively. In unit-vector notation, what is the net gravitational force on sphere \(B\) due to the other spheres?

In Problem 77, remove sphere \(A\) and calculate the gravitational potential energy of the remaining three-particle system. (b) If \(A\) is then put back in place, is the potential energy of the four-particle system more or less than that of the system in (a)? (c) In (a), is the work done by you to remove \(A\) positive or negative? (d) In (b), is the work done by you to replace \(A\) positive or negative?

A certain triple-star system consists of two stars, each of mass \(m\), revolving in the same circular orbit of radius \(r\) around a central star of mass \(M\) (Fig. 13-53). The two orbiting stars are always at opposite ends of a diameter of the orbit. Derive an expression for the period of revolution of the stars.

The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator just barely provides the centripetal force needed for the rotation. (Why?) (a) Show that the corresponding shortest period of rotation is

\[ T = \sqrt{\frac{3\pi}{Gp}}. \]
where $\rho$ is the uniform density (mass per unit volume) of the spherical planet. (b) Calculate the rotation period assuming a density of $3.0 \text{ g/cm}^3$, typical of many planets, satellites, and asteroids. No astronomical object has ever been found to be spinning with a period shorter than that determined by this analysis.

81 SSM In a double-star system, two stars of mass $3.0 \times 10^{30} \text{ kg}$ each rotate about the system's center of mass at radius $1.0 \times 10^{11} \text{ m}$. (a) What is their common angular speed? (b) If a meteoroid passes through the system's center of mass perpendicular to their orbital plane, what minimum speed must it have at the center of mass if it is to escape to “infinity” from the two-star system?

82A satellite is in an elliptical orbit with a period of $8.00 \times 10^4 \text{ s}$ about a planet of mass $7.00 \times 10^{24} \text{ kg}$. At aphelion, at radius $4.5 \times 10^7 \text{ m}$, the satellite's angular speed is $7.158 \times 10^{-5} \text{ rad/s}$. What is its angular speed at perihelion?

83 SSM In a shuttle craft of mass $m = 3000 \text{ kg}$, Captain Janeway orbits a planet of mass $M = 9.50 \times 10^{25} \text{ kg}$, in a circular orbit of radius $r = 4.20 \times 10^7 \text{ m}$. (a) What are the period of the orbit and (b) the speed of the shuttle craft? Janeway briefly fires a forward-pointing thruster, reducing her speed by 2.00%. Just then, what are (c) the speed, (d) the kinetic energy, (e) the gravitational potential energy, and (f) the mechanical energy of the shuttle craft? (g) What is the semimajor axis of the elliptical orbit now taken by the craft? (h) What is the difference between the period of the original circular orbit and that of the new elliptical orbit? (i) Which orbit has the smaller period?

84 A uniform solid sphere of radius $R$ produces a gravitational acceleration of $a_g$ on its surface. At what distance from the sphere's center are there points (a) inside and (b) outside the sphere where the gravitational acceleration is $a_g/3$?

85 ILW A projectile is fired vertically from Earth's surface with an initial speed of 10 km/s. Neglecting air drag, how far above the surface of Earth will it go?

86 An object lying on Earth's equator is accelerated (a) toward the center of Earth because Earth rotates, (b) toward the Sun because Earth revolves around the Sun in an almost circular orbit, and (c) toward the center of our galaxy because the Sun moves around the galactic center. For the latter, the period is $2.5 \times 10^8 \text{ y}$ and the radius is $2.2 \times 10^{20} \text{ m}$. Calculate these three accelerations as multiples of $g = 9.8 \text{ m/s}^2$.

87 (a) If the legendary apple of Newton could be released from rest at a height of 2 m from the surface of a neutron star with a mass 1.5 times that of our Sun and a radius of 20 km, what would be the apple's speed when it reached the surface of the star? (b) If the apple could rest on the surface of the star, what would be the approximate difference between the gravitational acceleration at the top and at the bottom of the apple? (Choose a reasonable size for an apple; the answer indicates that an apple would never survive near a neutron star.)

88 With what speed would mail pass through the center of Earth if falling in a tunnel through the center?

89 SSM The orbit of Earth around the Sun is almost circular: The closest and farthest distances are $1.47 \times 10^8 \text{ km}$ and $1.52 \times 10^8 \text{ km}$ respectively. Determine the corresponding variations in (a) total energy, (b) gravitational potential energy, (c) kinetic energy, and (d) orbital speed. (Hint: Use conservation of energy and conservation of angular momentum.)

90 A 50 kg satellite circles planet Cruton every 6.0 h. The magnitude of the gravitational force exerted on the satellite by Cruton is 80 N. (a) What is the radius of the orbit? (b) What is the kinetic energy of the satellite? (c) What is the mass of planet Cruton?

91 We watch two identical astronomical bodies $A$ and $B$, each of mass $m$, fall toward each other from rest because of the gravitational force on each from the other. Their initial center-to-center separation is $R_i$. Assume that we are in an inertial reference frame that is stationary with respect to the center of mass of this two-body system. Use the principle of conservation of mechanical energy ($K_i + U_i = K_f + U_f$) to find the following when the center-to-center separation is $0.5R_i$: (a) the total kinetic energy of the system, (b) the kinetic energy of each body, (c) the speed of each body relative to us, and (d) the speed of body $B$ relative to body $A$.

Next assume that we are in a reference frame attached to body $A$ (we ride on the body). Now we see body $B$ fall from rest toward us. From this reference frame, again use $K_f + U_f = K_i + U_i$ to find the following when the center-to-center separation is $0.5R_i$: (e) the kinetic energy of body $B$ and (f) the speed of body $B$ relative to body $A$. (g) Why are the answers to (d) and (f) different? Which answer is correct?

92A 150.0 kg rocket moving radially outward from Earth has a speed of 3.70 km/s when its engine shuts off 200 km above Earth's surface. (a) Assuming negligible air drag, find the rocket's kinetic energy when the rocket is 1000 km above Earth's surface. (b) What maximum height above the surface is reached by the rocket?

93 Planet Roton, with a mass of $7.0 \times 10^{24} \text{ kg}$ and a radius of 1600 km, gravitationally attracts a meteorite that is initially at rest relative to the planet, at a distance great enough to take as infinite. The meteorite falls toward the planet. Assuming the planet is airless, find the speed of the meteorite when it reaches the planet's surface.
Two 20 kg spheres are fixed in place on a y axis, one at y = 0.40 m and the other at y = -0.40 m. A 10 kg ball is then released from rest at a point on the x axis that is at a great distance (effectively infinite) from the spheres. If the only forces acting on the ball are the gravitational forces from the spheres, then when the ball reaches the (x, y) point (0.30 m, 0), what are (a) its kinetic energy and (b) the net force on it from the spheres, in unit-vector notation?

Sphere A with mass 80 kg is located at the origin of an xy coordinate system; sphere B with mass 60 kg is located at coordinates (0.25 m, 0); sphere C with mass 0.20 kg is located in the first quadrant 0.20 m from A and 0.15 m from B. In unit-vector notation, what is the gravitational force on C due to A and B?

In his 1865 science fiction novel *From the Earth to the Moon*, Jules Verne described how three astronauts are shot to the Moon by means of a huge gun. According to Verne, the aluminum capsule containing the astronauts is accelerated by ignition of nitrocellulose to a speed of 11 km/s along the gun barrel’s length of 220 m. (a) In g units, what is the average acceleration of the capsule and astronauts in the gun barrel? (b) Is that acceleration tolerable or deadly to the astronauts?

A modern version of such gun-launched spacecraft (although without passengers) has been proposed. In this modern version, called the SHARP (Super High Altitude Research Project) gun, ignition of methane and air shoves a piston along the gun’s tube, compressing hydrogen gas that then launches a rocket. During this launch, the rocket moves 3.5 km and reaches a speed of 7.0 km/s. Once launched, the rocket can be fired to gain additional speed. (c) In g units, what would be the average acceleration of the rocket within the launcher? (d) How much additional speed is needed (via the rocket engine) if the rocket is to orbit Earth at an altitude of 700 km?

An object of mass \( m \) is initially held in place at radial distance \( r = 3R_E \) from the center of Earth, where \( R_E \) is the radius of Earth. Let \( M_E \) be the mass of Earth. A force is applied to the object to move it to a radial distance \( r = 4R_E \), where it again is held in place. Calculate the work done by the applied force during the move by integrating the force magnitude.